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KEY
—
KATANAOKA'S
ARITHMETIC.

02
e. 129

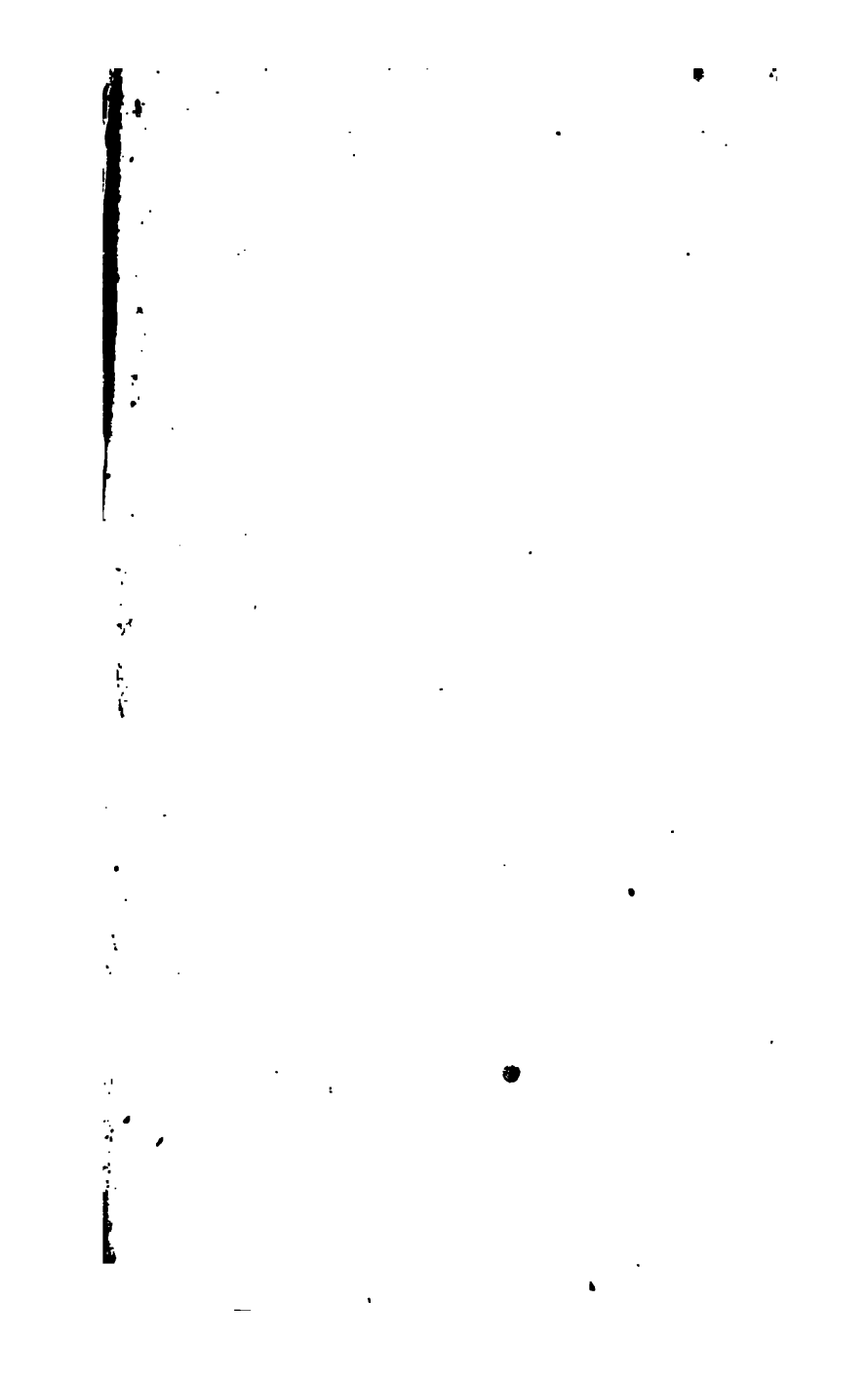


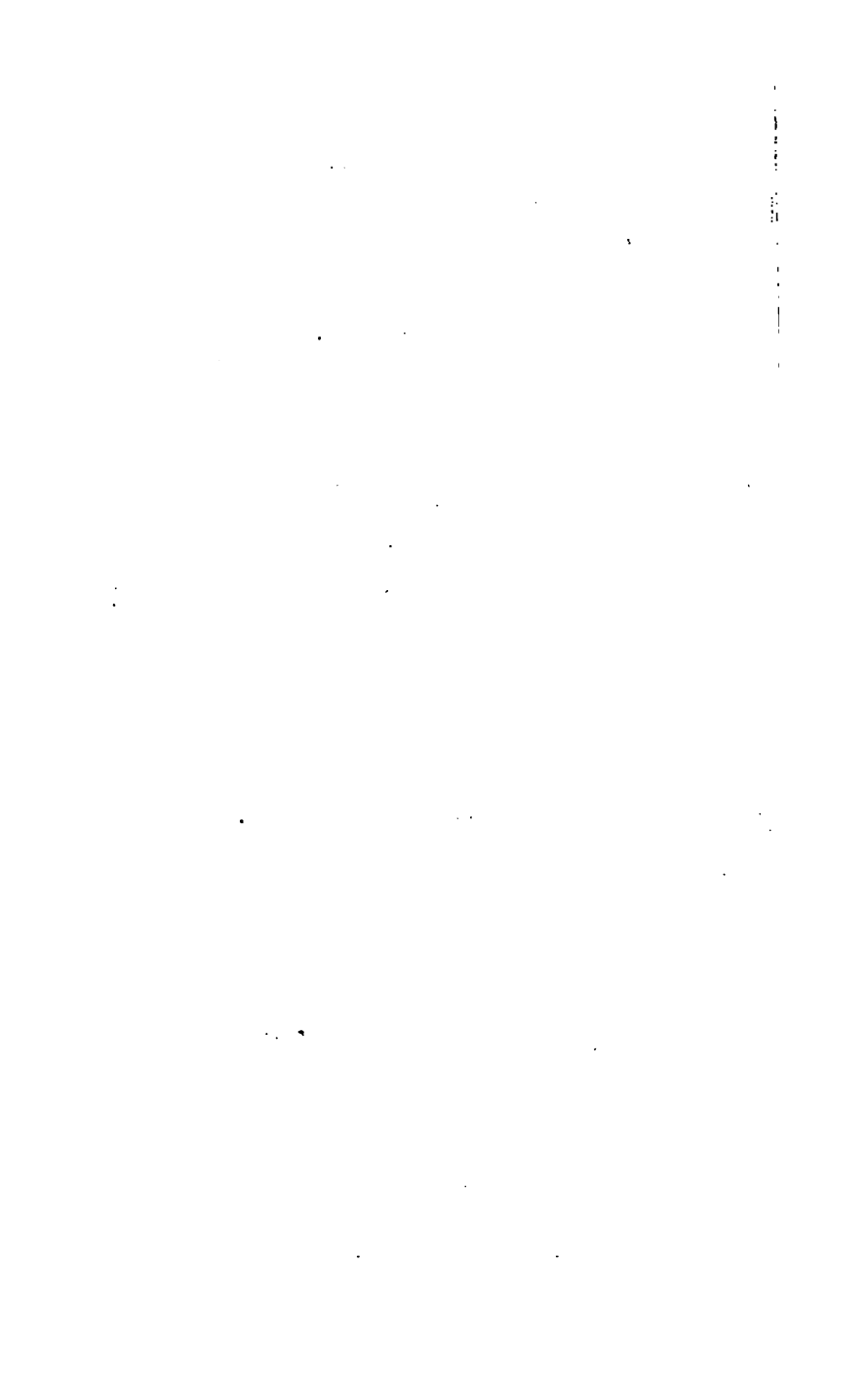
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K E Y

TO

KAVANAGH'S ARITHMETIC,

ITS

PRINCIPLES AND PRACTICE ;

ADAPTED TO THE SECOND, REVISED, AND SUBSEQUENT STEREOTYPE EDITIONS.

By THOMAS GILSON.

DUBLIN :

SAMUEL J. MACHEN, 28, WESTMORELAND-STREET.

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PREFACE.

It has long been a very general opinion amongst the less educated and enlightened classes of the community, that a Teacher's using a Key to Arithmetic, either publicly or privately in his school, was a proof of his inability and want of qualification for his office: but happily for the advancement of education, enlightenment, and consequent civilization, that absurd opinion is rapidly vanishing. In Endowed, National, and other Schools for the education of the poor, where one Teacher has to educate and instruct a hundred, and sometimes several hundred children, the use of a Key is indispensably necessary to enable him to perform his duty efficiently; as, by means of the Key he can see, at a glance, whether the pupil be correct in his operations, without the tedious labour of examining each operation, and thus save much time which can be more profitably employed in the other duties of his office. In this respect, the use of a Key should never be considered as discreditable to the Teacher.

The following pages, which are intended exclusively for the use of Teachers and those studying without the aid of a master, exhibit the working of all the questions and exercises in the Revised Edition of KAVANAGH'S ARITHMETIC, except a few of the more simple kind. The method of performing each operation is shewn, and the result of each step, in every process, is given. In the more advanced rules the operation is given at full length, by which the Teacher, it is to be expected, will derive much benefit from the unreserved use of the Key in pursuing the duties of his daily avocation.

Glasnevin, November, 1848.

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ERRATA.

Page 5, *Ezer.* 61, for 5—5 read 5—3.
Page 81, *Ezer.* 34, line 9 from top, for 3430390, &c., read 34303000, &c.; as
as root or answer, read (·024202).

KEY
TO
KAVANAGH'S ARITHMETIC
BY GILSON.

SIMPLE DIVISION.

(Arithmetic, page 41.)

If the teacher require the pupil to read out the remainder in each operation, he (the teacher) will be able to judge of its correctness, as all the remainders except the last are given underneath.

Exer.

10. 19, 49, 64.
11. 20, 11, 51, 11.
12. 19, 1.
13. 5, 14, 4, 7.
14. 28, 89, 33.
15. 17, 14, 37, 50.
16. 34, 99, 69, 74, 123.
17. 201, 176, 537, 478.
18. 37, 308, 540.
19. 42, 277.
20. 463, 694, 589, 247, 360.
21. 1204, 415, 274, 811, 358, 1649.
22. 5588, 1219.
23. 1922, 4790, 4605, 2761.
24. 1777, 1482.
25. 5062, 1246, 2589.
26. 2555, 1513, 708.
27. 921, 2004, 2018.

The following are solved by factors:

Exer.

28. $6)970488 \div 36$ factors 6×6

$$\begin{array}{r} 6)161748 \\ \hline 26958 \end{array}$$

29. $12)8075068 \div 132$, factors 12×11

$$\begin{array}{r} 11)672922 + 4 = 4 \text{ ones} = 4 \\ \hline 61174 + 8 = 8 \text{ twelves} = 96 \\ \hline 100 \text{ rem.} \end{array}$$

30. $2)218370 \div 288$, factors $2 \times 12 \times 12$

$$\begin{array}{r} 12)109185 \\ \hline 12)9098 + 9 = 9 \text{ twos} = 18 \\ \hline 758 + 2 = 2 \text{ twenty-fours} = 48 \\ \hline 66 \text{ rem.} \end{array}$$

31.

$12)90007 \div 432$, factors $12 \times 12 \times 3$

$$12)7500 + 7 = 7 \text{ ones} = 7$$

$$\begin{array}{r} 3)625 \\ \hline 208 + 1 = 1 \text{ one hundred \& forty-four} = 144 \\ \hline 151 \text{ rem.} \end{array}$$

32. $12)5080936 \div 108$, factors 12×9

$$9)423411 + 4 = 4 \text{ ones} = 4$$

$$\begin{array}{r} 47045 + 6 = 6 \text{ twelves} = 72 \\ \hline 76 \text{ rem.} \end{array}$$

33.

$12)3815643 \div 576$, factors $12 \times 12 \times 4$

$$12)317970 + 3 = 3 \text{ ones} = 3$$

$$4)26497 + 6 = 6 \text{ twelves} = 72$$

$$\begin{array}{r} 6624 + 1 = 1 \text{ one hundred \& forty-four} = 144 \\ \hline 219 \text{ rem.} \end{array}$$

SIMPLE DIVISION.

3

Exer.

$$\begin{array}{l}
 34. \quad 7 \overline{)4312438} \div 49, \text{ factors } 7 \times 7 \\
 \quad \quad 7 \overline{)616062} + 4 = 4 \text{ ones} = 4 \\
 \quad \quad \quad 88008 + 6 = 6 \text{ sevens} = \underline{42} \\
 \quad \quad \quad \quad \quad \quad 46 \text{ rem.}
 \end{array}$$

$$\begin{array}{l}
 35. \quad 11 \overline{)713085} \div 198, \text{ factors } 11 \times 9 \times 2 \\
 \quad \quad 9 \overline{)64825} + 10 = 10 \text{ ones} = 10 \\
 \quad \quad 2 \overline{)7202} + 7 = 7 \text{ elevens} = \underline{77} \\
 \quad \quad \quad 3601 \quad \quad \quad 87 \text{ rem.}
 \end{array}$$

$$\begin{array}{l}
 36. \quad 5 \overline{)58073} \div 125, \text{ factors } 5 \times 5 \times 5 \\
 \quad \quad 5 \overline{)11614} + 3 = 3 \text{ ones} = 3 \\
 \quad \quad 5 \overline{)2322} + 4 = 4 \text{ fives} = 20 \\
 \quad \quad \quad 404 + 2 = 2 \text{ twenty-fives} = \underline{50} \\
 \quad \quad \quad \quad \quad \quad 73 \text{ rem.}
 \end{array}$$

The Author considers the following too difficult for children to perform by factors, and therefore gives the solutions in the usual way.

$$37. \quad 20129, 49178, 35434, 50107, 44725, 41602, 10377.$$

$$38. \quad 1092, 2747, 2945, 4930.$$

$$39. \quad 1570, 7742, 5804, 2337, 7456, 2945, 5576.$$

$$40. \quad 22543, 5522, 23809.$$

$$41. \quad 260534, 306354, 107704.$$

$$42. \quad 3769, 1043, 5196, 4837, 1254, 2071, 5006.$$

$$43. \quad 49137, 87330, 65220, 46148, 57440.$$

$$44. \quad 721233, 713482, 525973.$$

$$45. \quad 439, 1476, 1808, 587.$$

$$46. \quad 58876, 48712, 37078.$$

$$47. \quad 591665, 477618.$$

$$48. \quad 28367, 30036, 15017, 23359.$$

$$49. \quad 7840, 8360, 4799, 4216, 7145, 1410.$$

$$50. \quad 4858, 11551, 4405, 7015, 8425.$$

SIMPLE DIVISION.

Exer.

51. From £177 × 2 = 354

Take 177 ÷ 3 = 59

£295

From £295

Take 109

186 then

£186 × 5 = 930 and

From £930

Take 177

£753 ÷ 3 = £251 for each.

$$\frac{([177 \times 2 - (177 \div 3) - 109] \times 5) - 177}{3} = £251.$$

52. $7 \times 7 \times 7 \times 7 \times 7 \times 4 = 67228$ from

$9 \times 9 + 4 = 85$ take

67143

$7 \times 7 \times 7 - 3 \times 3 \times 3 = 316$ and

$67143 \div 316 = 212\frac{1}{4}$

53.

$55000000 \div 1873000 = 29$ days and 683000 over.

54.

$\frac{23 \times 23 \times 23}{2 \times 2 \times 2} = 12167 \div 8 = 1520\frac{7}{8}$ Jupiter.

$\frac{19 \times 19 \times 19}{2 \times 2 \times 2} = 6859 \div 8 = 857\frac{3}{8}$ Saturn.

$\frac{17 \times 17 \times 17}{4 \times 4 \times 4} = 4913 \div 64 = 76\frac{5}{8}$ Herschel.

$112 \times 112 \times 112 = 1404928$ Sun.

55. $25020 \div 24 = 1042$ miles and 12 over.

56. $355320 \div 2912 = 122$ and 56 over.

57. $22269498 \div 193 = 115386$.

58. $9 \times 9 \times 9 \times 9 \times 9 = 59049$

$7 \times 7 \times 7 \times 7 \times 7 \times 7 = 117649$

Difference = 58600

$12 \times 12 \times 12 \times 12 \times 12 = 248832 \times 6 \times 6 \times 6 = 53747712$

then $53747712 \div 58600 = 917$ and 11512 over.

Exer.

59.

$$\overline{8 \times 8 \times 8} - \overline{5 \times 5} = 512 - 25 = 487 \text{ difference.}$$

$$512 + 25 = 537 \text{ sum.}$$

$$512 \times 25 = 12800 \text{ product.}$$

$$512 \div 25 = 20 + 12 \text{ quotient.}$$

Total sum $13844 + 12$ Answ.

$$60. \left. \begin{array}{l} 18526567 \div 89979 = 205 + 80872 \text{ over} \\ 8205382 \div 31874 = 257 + 13764 \text{ over} \end{array} \right\} \text{Ans.}$$

$$61. \quad \overbrace{5-5} = 2, \text{ the index of the quotient,} \\ \text{then } 18 \times 18 = 324 \text{ or } 18^2 \text{ Ans.}$$

$$62. \quad \overbrace{9-7} = 2, \text{ the index of the quotient,} \\ \text{then } 96 \times 96 = 9216 \text{ or } 96^2 \text{ Ans.}$$

$$63. (15 \times 9)^3 = 135^3 \text{ and } \overline{3 \times 3 \times 3 \times 5^3} = 135^3 \text{ and}$$

$$\overbrace{5-2} = 3, \text{ the index of the quotient,} \\ \text{then } 135 \times 135 \times 135 = 2460375 \text{ or } 135^3 \text{ Ans.}$$

$$64. (\frac{8}{7})^8 = 8^8, \text{ then } \overbrace{8-5} = 3, \text{ index of the quotient,} \\ \text{then } 8 \times 8 \times 8 = 512 \text{ or } 8^3 \text{ Ans.}$$

$$65. \quad (24 \times 9)^4 = 216^4 \\ (12 \times 2 \times 3^2)^4 = (12 \times 2 \times 3 \times 3)^4 = 216^4 \\ \text{and } 216^4 \div 216^4 = 1, \text{ Ans.}$$

$$66. \left(\frac{24+11-15}{2} \right)^{10} = \left(\frac{35-15}{2} \right)^{10} = \left(\frac{20}{2} \right)^{10} = 10^{10}$$

$$(2 \times 5)^9 = 10^9 \text{ and } \overbrace{10-9} = 1, \text{ index of the quotient,} \\ \text{therefore, } 10^1 \text{ or } 10 \text{ Ans.}$$

$$68. \quad \overbrace{11-7} = 4, \text{ index of quotient} = 19^4$$

$$\overbrace{27-25} = 2, \text{ index of quotient} = 19^2$$

$$\text{and } \overbrace{4-2} = 2, \text{ index of quotient} = \\ \text{then } 19 \times 19 = 361, \text{ or } 19^2 \text{ Ans.}$$

SIMPLE DIVISION.

Exer.

67.

$$\overline{17 \times 17 \times 17 \times 17} = 83521$$

$$\overline{11 \times 11 \times 11} = 1331$$

$$\text{difference} = \overline{82190^7}$$

$$\overline{12 \times 12 \times 12 \times 12} = 20736 \times 4 = 82944$$

$$2 \times 377 = \overline{754}$$

$$\text{difference} = \overline{82190^5}$$

then $\overbrace{7 - 5} = 2$, index of the quotient,
and $82190 \times 82190 = 6755196100$, or 82190^2 Ans.

$$69. \quad (24 - 11 + 16)^5 = (13 + 16)^5 = 29^5$$

$$(\overbrace{7 \times 4 + 1}^8)^8 = (28 + 1)^8 = 29^8$$

and $\overbrace{8 - 5} = 3$, index of quotient,
then $29 \times 29 \times 29 = 24389$, or 29^3 Ans.

$$70. \quad 10^{(3+2)} = 10_5$$

$$10^{(2 \times 2)} = 10_4$$

and $\overbrace{5 - 4} = 1$, index of quotient,
then 10, or 10^1 Ans.

EVOLUTION.

(Arithmetic, page 49.)

71.

$$40,76 \overset{\text{root.}}{(63 \text{ Ans.})}$$

$$36$$

$$123 \overline{)476}$$

$$369$$

$$\overline{107 \text{ rem.}}$$

72.

$$21,35,43 \overset{\text{root.}}{(462 \text{ Ans.})}$$

$$16$$

$$86 \overline{)535}$$

$$516$$

$$922 \overline{)1943}$$

$$1844$$

$$\overline{99 \text{ rem.}}$$

Exer.

$$\begin{array}{r}
 73. \qquad 51,48 \overset{\text{root.}}{(71 \text{ Ans.}} \\
 \underline{49} \\
 141 \overline{)248} \\
 \underline{141} \\
 107 \text{ rem.}
 \end{array}$$

$$\begin{array}{r}
 74. \\
 (54 \times 7 + 28 \times 9)^{\frac{1}{2}} = 378 + 252^{\frac{1}{2}} = \overset{\text{root.}}{\sqrt{6,30}} (25 \text{ Ans.} \\
 \underline{4} \\
 45 \overline{)230} \\
 \underline{225} \\
 5 \text{ rem.}
 \end{array}$$

$$\begin{array}{r}
 75. \ 52 \times 52 = 2704, \ 30 \times 30 \times 30 \times 30 = 810000 \\
 \text{from } 810000 \\
 \text{take } \underline{2704} \\
 \text{difference} = 807296 \\
 11 \times 11 \times 11 = \underline{1331} \\
 \overset{\text{root.}}{80,86,27} (899 \text{ Ans.} \\
 \underline{64} \\
 169 \overline{)1686} \\
 \underline{1521} \\
 1789 \overline{)16527} \\
 \underline{16101} \\
 426 \text{ rem.}
 \end{array}$$

$$\begin{array}{r}
 76. \quad \sim \\
 3-1=2, \text{ index of quotient,} \\
 \text{then } 36 \times 36 = 1296, \text{ or } 36^2 \\
 \sim \\
 4-2=2, \text{ index of quotient,} \\
 \text{then } 15 \times 15 = 225, \text{ or } 15^2 \overset{\text{root.}}{} \\
 \text{again } 1296 \div 225 = 15,21 (39 \text{ Ans.} \\
 \underline{9} \\
 69 \overline{)621} \\
 \underline{621}
 \end{array}$$

Exer.

77.

$$(27 \times 9 + 18 \times 27^2)^{\frac{1}{4}} = \overline{27 \times 27 \times 27 \times 27}^{\frac{1}{4}} = 53,14,41 \begin{matrix} \text{root.} \\ 729 \\ \text{Ans.} \end{matrix}$$

$$\begin{array}{r} 49 \\ 142 \overline{)414} \\ \underline{284} \\ 1449 \overline{)13041} \\ \underline{13041} \end{array}$$

78. $8 \div 2 = 4$, index of required root.
therefore $154^4 = 154 \times 154 \times 154 \times 154 = 562448656$ Ans.

79. $\begin{matrix} \text{root.} \\ 36 \end{matrix} (6 \text{ Ans.})$

$$\begin{array}{r} 36 \\ \hline \end{array}$$

80. $52 \times 24 = 1248$
 $144 \times 7 = 1008$ $\begin{matrix} \text{root.} \\ 2,40 \end{matrix} (15 \text{ Ans.})$

$$\begin{array}{r} 1 \\ 25 \overline{)140} \\ \underline{125} \\ 15 \text{ rem.} \end{array}$$

THIRD ROOT.

(Arithmetic, page 51).

81. $\begin{matrix} \text{root.} \\ 1,728 \end{matrix} (12 \text{ Ans.})$

$$\begin{array}{r} 1 \\ \hline 728 \text{ dividend.} \end{array}$$

$$\begin{array}{r} 1^2 \times 3 = 3 \dots \\ 1 \times 2 \times 3 = 6 \dots \\ 2^2 = 4 \end{array} \left| \begin{array}{l} \\ \\ \end{array} \right.$$

$$2 \times 364 = 728$$

$$\dots 0 \text{ rem.}$$

Exer.

82. $5,621,743$ (177 ^{root.} Ans.)

$$\begin{array}{r}
 1 \\
 \hline
 4621 \text{ dividend.} \\
 1^2 \times 3 = 3.. \\
 1 \times 3 \times 7 = 21. \\
 7^2 = 49 \\
 \hline
 7 \times 559 = 3913 \\
 \hline
 708743 = 2\text{nd dividend.} \\
 17^2 = 289 \times 3 = 867.. \\
 17 \times 3 \times 7 = 357. \\
 7^2 = 49 \\
 \hline
 7 \times 90319 = 632233 \\
 \hline
 76510 \text{ rem.}
 \end{array}$$

83. $8,000,000$ (200 ^{root.} Ans.)

$$\begin{array}{r}
 8 \\
 \hline
 000,000 \text{ rem.}
 \end{array}$$

85. $23^2 \times 15^2 = 529 \times 3375 = 1,785,375$ (121 ^{root.} Ans.)

$$\begin{array}{r}
 1 \\
 \hline
 785 \\
 1^2 \times 3 = 3.. \\
 1 \times 3 \times 2 = 6. \\
 2^2 = 4 \\
 \hline
 2 \times 364 = 728 \\
 \hline
 57375 \\
 12^2 \times 3 = 432.. \\
 12 \times 3 \times 1 = 36. \\
 1^2 = 1 \\
 \hline
 1 \times 43561 = 43561 \\
 \hline
 13814 \text{ rem.}
 \end{array}$$

86. $27 \times 9 - 57 \div 19 = 243 - 3 = 240$ (^{root.} Ans.)

EVOLUTION.

Exer.

84.

$$\begin{array}{r} \text{root.} \\ 432,165 \text{ (75 Ans.} \\ \underline{343} \end{array}$$

89165 dividend.

$$\begin{array}{r} 7 \times 3 = 147 \dots \\ 7 \times 3 \times 5 = 105 \dots \\ \underline{5^2 = 25} \end{array}$$

$$5 \times 15775 = 78875$$

10290 rem.

87.

$$\begin{array}{r} \text{root.} \\ 729 \times 28 \div 14 = 1,458 \text{ (11 Ans.} \\ \underline{1} \end{array}$$

$$\begin{array}{r} 1^2 \times 3 = 3 \dots 458 \\ 1 \times 1 \times 3 = 3 \dots \\ \underline{1^2 = 1} \end{array}$$

$$1 \times 331 = 331$$

127 rem.

89.

$$56 \times 56 \times 56 \times 56 \times 56 \times 56 = 30840979456$$

$$28 \times 28 \times 28 = 21952$$

$$\begin{array}{r} \text{root.} \\ 30,841,001,408 \text{ (3135} \\ \underline{27} \end{array} \text{ [Ans.}$$

$$\begin{array}{r} 3^2 \times 3 = 27 \dots 3841 \text{ dividend.} \\ 3 \times 1 \times 3 = 9 \dots \\ \underline{1^2 = 1} \end{array}$$

$$1 \times 2791 = 2791$$

$$\begin{array}{r} 31^2 \times 3 = 2883 \dots 1050001 \text{ dividend.} \\ 31 \times 3 \times 3 = 279 \dots \end{array}$$

$$3^2 = 9$$

$$3 \times 291099 = 873297$$

$$\begin{array}{r} 313^2 \times 3 = 293907 \dots 176704408 \text{ dividend.} \\ 313 \times 5 \times 3 = 4695 \dots \end{array}$$

$$5^2 = 25$$

$$5 \times 29437675 = 147188375$$

29516033 rem.

Exer.

88.

then $2\sqrt{144}=12$
 $12 \times 12^4 = 12^5 = 12 \times 12 \times 12 \times 12 \times 12 = 248,832$ (62 Ans. root.)

$$\begin{array}{r}
 6^2 \times 3 = 108 \dots 32832 \\
 6 \times 2 \times 3 = 36 \dots 216 \\
 2^2 = 4 \dots \\
 \hline
 2 \times 11164 = 22328 \\
 10504 \text{ rem.}
 \end{array}$$

90.

12,564,217 (232 Ans. root.)
 8

$$\begin{array}{r}
 2^2 \times 3 = 12 \dots 4564 \text{ dividend.} \\
 2 \times 3 \times 3 = 18 \dots \\
 3^2 = 9 \dots \\
 \hline
 3 \times 1389 = 4167 \\
 23^2 \times 3 = 1587 \dots 397217 \text{ dividend.} \\
 23 \times 2 \times 3 = 138 \dots \\
 2^2 = 4 \dots \\
 \hline
 2 \times 160084 = 320168 \\
 77049 \text{ rem.}
 \end{array}$$

91. 24,13,75,69⁴

$$\begin{array}{r}
 16 \\
 89 \overline{)813} \quad 1^2 \times 3 = 3 \dots 3913 \\
 \underline{801} \quad 1 \times 7 \times 3 = 21 \dots \\
 981 \overline{)1275} \quad 7^2 = 49 \dots \\
 \underline{981} \quad 7 \times 559 = 3913 \\
 9823 \overline{)29469} \quad 0 \text{ Rem.} \\
 \underline{29469}
 \end{array}$$

93.

4,096

$$\begin{array}{r}
 1 \\
 1^2 \times 3 = 3 \dots 3096 \\
 1 \times 6 \times 3 = 18 \dots \\
 6^2 = 36 \dots \\
 \hline
 6 \times 516 = 3096
 \end{array}$$

$\left\{ \begin{array}{l} 2\sqrt{16} = 2\sqrt{4} = 2 = [(2^2)^2]^2 \\ \underline{16} \quad \underline{4} \end{array} \right.$ (Ans. root.)

Exer.

92.

3,814,697,265,625

1

$$1^2 \times 3 = 3 \dots | \quad 2814 \text{ dividend.}$$

$$1 \times 5 \times 3 = 15 \dots |$$

$$5^2 = 25 \quad |$$

$$\underline{5 \times 475 = 2375}$$

$$15^2 \times 3 = 675 \dots | \quad 439697 \text{ dividend.}$$

$$15 \times 6 \times 3 = 270 \dots |$$

$$6^2 = 36 \quad |$$

$$\underline{6 \times 70236 = 421416}$$

$$156^2 \times 3 = 73008 \dots | \quad 18281265 \text{ dividend.}$$

$$156 \times 2 \times 3 = 966 \dots |$$

$$2^2 = 4 \quad |$$

$$\underline{2 \times 7310164 = 14620328}$$

$$1562^2 \times 3 = 7319532 \dots | \quad 3660937625 \text{ dividnd.}$$

$$1562 \times 5 \times 3 = 23430 \dots |$$

$$5^2 = 25 \quad |$$

$$\underline{5 \times 732187525 = 3660937625}$$

3rd root.

15,625

root.

(25 = (25²)³ Ans.

8

$$2^2 \times 3 = 12 \dots | \quad 7625$$

$$2 \times 5 \times 3 = 30 \dots |$$

$$5^2 = 25 \quad |$$

$$\underline{5 \times 1525 = 7625}$$

0 rem.

94.

$\overbrace{3-2=1}$, index of quotient,
then $216^1 = 216$ and

$$\sqrt[3]{216} \quad | \quad \text{root.}$$

$$216 \quad | \quad 6 \text{ Ans.}$$

0 rem.

Exer.

		2nd root.	4th root.	8th root.
95.	14,75,78,90,56	(3,84,16	(1,96	(14 Ans.
	<u>9</u>	<u>1</u>	<u>1</u>	
	68)575	29)284	24)96	
	<u>544</u>	<u>261</u>	<u>96</u>	
	764)3178	386)2316		
	<u>3056</u>	<u>2316</u>		
	7681)12290			
	<u>7681</u>			
	76826)460956			
	<u>460956</u>			

		2nd root.	4th root.
96.	1,67,96,16	(12,96	(36 Ans.
	<u>1</u>	<u>9</u>	
	22)67	66)396	
	<u>44</u>	<u>396</u>	
	249)2396		
	<u>2241</u>		
	2586)15516		
	<u>15516</u>		

REDUCTION.

(Arithmetic, p. 73.)

1. 1120*s*.
2. 4644*d*.
6. 2200*s*. 26404*d*.
7. 239*d*.
11. 4937*d*. 411*s*.
13. 8041*d*. 670*s*.
14. 3819*d*.
15. 7869*s*. 94436*d*.
16. 146*s*.

Exer.

17. 102776*d*.
19. 7740*s*.
20. 14742*s*.
21. 10710*d*.
22. 4918*d*.
23. 36225*s*.
24. $856 \times 30 = 25680*d*. \div 12 = 2140*s*.$
25. 200160 fur. 8006400 per. 44035200 yds.
27. 24594 yds. 4471 per. 111 fur.
29. 463 fur. 18540 per. 101973 yds. 305913 feet.
30. 3215 per. 80 fur.
31. 707 rds.
32. 307200 per. 7680 rds. 1920 acres.
33. 2292 rds. 91680 per. 2773320 yds. 24959880 ft.
34. 12 rds.
37. 700 quarts.
38. 1548 pecks, 3096 gals.
39. 348 pints, 174 quarts.
40. 2467 gals.
41. 2500 cwt. 10000 qrs.
42. 14 qrs. 407 lbs.
43. 119 cwt. 477 qrs. 13379 lbs. 214064 ozs.
44. 37317 lbs. 1332 qrs. 333 cwts.
45. 621 qrs.
46. 210 oz. 4209 dwts.
47. 1932 oz.
48. 159 dwts.
49. 681 oz. Troy $\div 16$.
50. 2218464 oz. $\div 12$.
55. 9490 days, 227760 hours, 13665600 min.
56. 2894 hours, 120 days.
57. 2118 days.
59. 189 days, 4536 hours.
61. 200160 fur. 8006400 per. 44035200 yds,
132105600 ft.
62. 708 hours, 42524 min.
63. 16666 min. 277 hours.
64. 1157 qrs. 289 cwts.

Exer.

65. 48 fur. 1920 per. 10560 yds.

66. 480 dwt. $\div 29$.69. 120 fur. 4800 per. 26400 yds. 79200 ft. $\div 13$.70. 46 rds. 3055 per. $\div 6$ rds. or 259 per.

COMPOUND MULTIPLICATION.

(Arithmetic, page 78.)

<i>Exers.</i>		£	s.	d.	
19.	Prodt. by 6 =	119	7	9	$\times 3 =$ Answ.
20.	...	5	26	8	$2\frac{3}{4} \times 3 =$ "
21.	...	8	293	18	$10 \times 3 =$ "
22.	...	9	64	10	$9 \times 3 =$ "
23.	...	8	7	17	$0 \times 4 =$ "
24.	...	11	33	8	$11\frac{1}{2} \times 4 =$ "
25.	...	7	12	7	$7\frac{1}{2} \times 7 =$ "
26.	...	9	5	15	$6 \times 6 =$ "
27.	...	8	41	10	$2 \times 7 =$ "
28.	...	9	176	3	$1\frac{1}{2} \times 7 =$ "
29.	...	12	21	11	$3 \times 7 =$ "
30.	...	11	142	17	$3 \times 9 =$ "
31.	...	12	39	0	$9 \times 9 =$ "
32.	...	8	159	1	$8 \times 9 = 1431$ 15 0 $\times 3 =$ Answ.
33.	...	12	88	19	$0 \times 9 = 800$ 11 0 $\times 3 =$ "
34.	...	12	6	19	$0 \times 11 =$ Answ.
35.	...	8	237	19	$8 \times 7 = 1665$ 17 8 $\times 3 =$ "
36.	...	11	1	13	$8\frac{1}{2} \times 11 =$ Answ.
37.	...	8	24	6	$8 \times 8 = 194$ 13 4 $\times 8 =$ "
38.	...	4	7	10	$6 \times 4 = 30$ 2 0 $\times 7 =$ "
39.	...	11	10	13	$7 \times 7 = 74$ 15 1 $\times 3 =$ "
40.	...	5	27	6	$9\frac{1}{2} \times 5 = 136$ 13 $10\frac{1}{2} \times 5 =$ "

In the following solutions, where convenient factors cannot be found, factors are taken of a number somewhat greater or less than the given number; the product of those factors are given, with the remaining figure +, or —, whose product by the original multiplicand, added to, or taken from, the last product, will produce the required result.

<i>Exers.</i>		£	s.	d.	
41.	Prodt. by 4 =	31	17	1	$\times 4 = 127$ 8 4 $+ \times 1 =$ Ans.
42.	...	4	37	2	$2 \times 5 = 185$ 10 10 $- \times 1 =$ "
43.	...	4	47	10	$4 \times 6 = 285$ 2 0 $- \times 1 =$ "

COMPOUND DIVISION.

<i>Exer.</i>		£	s.	d.		£	s.	d.	
44.	Prodt. by 6 =	78	5	10½	× 5 =	391	9	4½	— × 1 = Ans.
45.	...	6	5	19	0 × 6 =	35	14	0	+ × 1 = "
46.	...	7	36	3	4 × 6 =	217	0	0	+ × 1 = "
47.	...	12	104	15	6 × 10 =	1047	15	0	+ × 7 = "
48.	...	10	76	1	8 × 10 =	760	16	8	× 3 = "
		2282	10	0	+ × 11 =	.	.	.	"
49.	...	8	7	5	2 × 10 =	72	11	8	— × 1 = "
50.	...	10	114	19	4½ × 11 =	1264	13	1½	+ × 1 = "
61.	...	5	1	3	11½ × 5 =	5	19	9½	× 7 = "
62.	...	12	10	7	6 × 11 =	114	2	6	— × 1 = "
63.	...	6	0	14	9 × 4 =	2	19	0	+ × ½ = "
64.	...	10	21	3	1½ × 5 =	105	15	7½	+ × 2 = "
65.	...	10	38	14	2 × 5 =	193	10	10	× 5 = "
		967	14	2	+ × 3 =	.	.	.	"
66.	...	5	1	19	7 × 9 =	17	16	3	× 7 = "
		124	13	9	— × ¾ =	.	.	.	"
67.	...	10	38	14	7 × 5 =	193	12	11	+ × 1 = "
68.	...	5	0	9	8½ × 5 =	2	8	5½	× 7 = "
69.	...	10	5	12	1 × 5 =	28	0	5	× 3 = "
		84	1	3	+ × 1 =	.	.	.	"
70.	...	10	0	17	1 × 9 =	7	13	9	+ × 1 = "

COMPOUND DIVISION.

(Arithmetic, page 84.)

The following solutions are performed by what is generally termed *long division*, as the solution by factors would involve fractions, with which the learner is, at this stage, supposed to be unacquainted. The remainder in each stage of the process is given, except the last, which is found in the arithmetic.

<i>Exer.</i>		<i>Exer.</i>
13.	£8 16s.	22. £68. 26s. 90d.
14.	£12 10s. 4s. 14d.	23. £317. 310s. 188s.
15.	£6. 12s.	312d.
16.	£32. 67s. 9d. 20d.	24. £27. 25s. 43d.
17.	£10. 28s. 16d.	26. 19 m. 19 fur. 21 p.
18.	£6. 10s. 20d.	28. 31 a. 17 rds. 16 p.
19.	£59. £15. 31s. 93d.	30. 18 cwt. 26 qrs. 26 lb.
20.	£48. 20s. 47s. 30d.	32. 13 dys. 51 hrs. 38 m.
21.	£51. 18s. 17s. 47d.	9 m.

<i>Exer.</i>		<i>Exer.</i>	
33. £2. 8s.		38. £156. 656s. 437d.	
34. £11. 26s. 4d. 22d.		39. £486. 720s. 1140d.	
35. £5. 24s. 17d.		40. £541. 1148s. 223d.	
36. £25. 31s. 28d.		41. £187. 101s. 125d.	
37. £41. 5s. 67d.		42. £4102285. 8197262s.	
		8107942d.	

43. { Britain, 2006859 a. 8027436 r. 13583177 p.
 { Ireland, 3988596 a. 7749002 r. 53798620 p.

44. £50000. 31666s.

45. £23 15s. 9d. $\times 4 = 95 \quad 3 \quad 0$

Interest on stock, $312 \quad 17 \quad 4\frac{1}{2}$

£17 10s. $3\frac{1}{2}d. \times 12 = 210 \quad 3 \quad 6$

Total, $618 \quad 3 \quad 10\frac{1}{2}$

Deduct annuity, $103 \quad 11 \quad 9\frac{3}{4}$

$365 \overline{) 514 \quad 12 \quad 0\frac{1}{4}}$

$\frac{365}{149} \quad [1 \quad 18 \quad 2\frac{1}{4} \quad \text{Answ.}$

$\frac{149}{20}$

$\frac{20}{2992}$

$\frac{2992}{2920}$

$\frac{2920}{72}$

$\frac{72}{12}$

$\frac{12}{864}$

$\frac{864}{730}$

$\frac{730}{134}$

$\frac{134}{4}$

$\frac{4}{539}$

$\frac{539}{365}$

$\frac{365}{174 \text{ remainder.}}$

SIMPLE PROPORTION.

(Arithmetic, page 95.)

Exer.

1. 15 cwt. : 10 cwt. :: £33 : £22.
2. £21 : £9 :: 14 p. : 6 p.
3. £21 : £28 :: 12 tons : 16 tons.
4. 182 cwt. : 126 cwt. :: £637 : £441.
5. £18 : £6 :: 24 tons : 8 tons.
6. £90 : £36 :: 25 acres : 10 acres.
7. 96 gals. : 57 gals. :: £64 : £38.
8. 24 men : 21 men :: 56 p. : 49 p.
9. £289 : £272 :: 153 cwt. : 144 cwt.
10. 15 cows : 65 cows :: £72 : £312.
11. 24 men : 18 men :: 12 days : 9 days.
12. £146 : £22 :: 365 days : 55 days.
13. 8 hours : 10 hours :: 12 days : 15 days.
14. 7 cwt. 1 qr. : 20 tons 6 cwt. :: £1 16 10½;
or 29 qrs. : 1624 :: 885 halfpence : 49560 halfpence, or £103 5 0.
15. £72 3 5½ : £20 12 5 :: 176 lbs. 10 st. 7 lbs.;
or 34643 halfp. : 9898 halfp. :: 34643 lbs. :
9898 lbs., or 50 bls. 7st.
16. £997 5 6¼ : £199 9 1¼ :: 257 acres 2 rds.;
or 957385 far. : 191477 far. :: 1030 rds. : 206 rds.
or 51 acres 2 rds.
17. 1 cwt. 2 qrs. 7 lbs. : 1 qr. 11 lbs. :: £41 18 6½;
or 175 lbs. : 39 lbs. :: 20125 halfpence : 4485
halfpence, or £9 6 10½.
18. 1 mile, 3 per. : 4 fur. 15 per. :: £31 5 9¾;
or 323 per. : 175 per. :: 30039 far. : 16275 far.,
or £16 19 0¾.
19. £3 17 5 : £17 8 4½ :: 2 cwt. 1 qr. 1 lb.;
or 1858 halfpence : 8361 halfpence :: 253 lbs. :
1858½ lbs., or 1 cwt. 0 qr. 18½ lbs.
20. £209 1 4 : 12s. 5¼d. :: 5 ton, 12 cwt.;
or 200704 farthings : 597 farthings :: 112 cwt. :
1 qr. 9 lbs. 5 oz.

Exer.

21. 39 men : 52 men :: 15 days : 20 days.
22. 15 bls. 19 st. 11 lbs. : 143 bls. 18 st. 1 lb. ::
 $\pounds 17\ 13\ 5\frac{1}{2}$;
 or 4477 lbs. : 40293 lbs. :: 16965 farthings :
 152685 farthings, or $\pounds 159\ 0\ 11\frac{1}{2}$.
23. 19s. 5d. : $\pounds 228\ 2\ 11$:: 1 yard.
 or 233d. : 54755d. :: 1 yard : 235 yards.
24. 1s. 8 $\frac{1}{2}$ d. : $\pounds 7\ 15\ 5\frac{1}{2}$:: 1 lb. ;
 or 41 halfp. : 3731 halfp. :: 1 lb. : 91 lbs.
25. 24 days : 32 days :: 18 men : 24 men.
26. 1 cwt. : 37 cwt. 2 qrs. 17 lbs. :: 15s. 8d. ;
 or 112 lbs. : 4217 lbs. :: 47d. : 7078 $\frac{1}{2}$ d., or
 $\pounds 29\ 9\ 10\frac{1}{2}$ and 4 over.
27. 1 doz. : 136 doz. 8 bottles :: $\pounds 3\ 4\ 6$.
 or 12 bots. : 1640 bots. :: 774d. : 105780d., or
 $\pounds 440\ 15\ 0$.
28. 13s. 4d. : $\pounds 96$:: 1 ream.
 or 160d. : 23040d. :: 1 ream : 144 reams.
29. 1 ton, 13 cwt. : 2 cwt. 3 qrs. 14 lbs. :: $\pounds 7\ 16\ 5$. ;
 or 3696 lbs. : 322 lbs. :: 1877d. : 163 $\frac{1}{2}$ d., or
 13s. 7 $\frac{1}{2}$ d. and 28 over.
30. 1 stone : 275 sacks :: 2s. 2 $\frac{1}{2}$ d. ;
 or 1 st. : 5500 st. :: 53 halfp. 291500 halfp., or
 $\pounds 607\ 5\ 10$.
31. 6 mile : 40 mile, 3 fur. 17 per. :: 14 min. 13 sec.
 or 1920 per. : 12937 per. 853 sec. : 5747 sec.,
 or 1 hour, 35 min. 47 sec. and 1021 over.
32. 1 lb. 15 grs. : 73 lbs. 9 oz. 18 dwt. :: $\pounds 1\ 18\ 7$;
 or 5775 grs. : 425232 grs. :: 463d. : 34092d.,
 or $\pounds 142\ 1\ 0$ and 1116 over.
33. 18s. 8d. : $\pounds 164\ 19\ 4$:: 1 acre ;
 or 224d. : 39592d. :: 1 acre : 176 acres, 3 rds.
34. 1 barrel : 257 barrels, 11 stone :: $\pounds 1\ 4\ 8$;
 or 12 stone : 3095 stone : 296d. : 76343 $\frac{1}{2}$ d., or
 $\pounds 318\ 1\ 11\frac{1}{2}$ and 1 over.
35. 29933 : 29833 :: 41847426 ft. : 41707622 ft.,
 or 7899 miles, 1 fur. 14 p. 6 yds. 2 feet, and
 10532 over.

Exer.

36. £1 17 6 : £1053 0 11½ :: 1 acre ;
 or 1800 farthings : 1010925 farthings :: 1 acre :
 561 acres, 2 rds. 20 perches.

37. 14 : 11 :: 15 ft. × 15 ft. : 176 ft. and 11 over.

38. 30 cwt. : 9 tons, 13 cwt. :: £116 13 6 ;
 or 30 cwt. : 193 cwt. :: 28002d. : 180146d.,
 £750 12 2, and 1 over.

39. 113 : 355 :: 41847426 ft. : 131467577 ft. equa-
 torial ; 113 : 355 :: 41706720 ft. : 131025536 ft.
 polar.

40. 15 × 15 = 225,	6 × 6 = 36,	7 × 7 × 7 = 343
9 × 9 = 81	8 × 8 = 64	10 × 10 = 100
difference, <u>144</u>	sum, <u>100</u>	sum, <u>443</u>
		less, <u>2</u>
		difference, <u>441</u>

then √144 : √100 :: √441 :: 17½ ;

or 12 : 10 :: 21 : 17½.

41.

s.	d.
From 12	6
take 11	3
<u>1</u>	<u>3</u>

then 1 gal. : 180 gals. :: 1s. 3d. : 225s. to be
 made up of water, at 11s. 3d. per gallon,
 and 11s. 3d. : 225s. :: 1 gal. ;
 or 135d. : 2700d. :: 1 gal. : 20 gals.

42. £1920 : £1 :: £410 17 6 ;
 or £1920 : £1 :: 98610d. : 51½d. or 4s. 3½d.

43. 9 journeys : 4 journeys :: 36 days : 16 days.

44. £26 15 0 : £100 :: £1 6 9 ;
 or 535s. : 2000s. :: 321d. : 1200d. or £5.

45. £17 3 5 : £60 1 11½ :: £136.
 or 8242 halfp. : 28847 halfp. :: £136 : £476.

46. 8 months : 9 months :: £56 : £63.

Exer.

47.

$$4^3 : 3^4 :: \begin{cases} 5 \times 5 = 25 \\ 3 \times 3 = 9 \\ \text{sum, } 34 \\ \text{less, } 2 \end{cases}$$

$$4 \times 4 \times 4 = 64 : 3 \times 3 \times 3 \times 3 = 81 :: 32 : 40\frac{1}{2}.$$

48. 1 bar. : 29 bls. 12 st. :: 9s. 2d.;

or 14 stone : 418 stone :: 110d. : 3284 $\frac{1}{4}$ d., or
£13 13 8 $\frac{1}{4}$.

Again, £2 12 6 : £13 13 8 $\frac{1}{4}$:: 1 cwt.;

or 2520 far. : 13137 far. :: 1 cwt. : 5 cwt., 0 qrs.
23 $\frac{1}{2}$ lbs. and 1848 over.

49. From 3816 gals. First cost, £216 13 0
charges, 34 15 6
take 132 $\frac{3}{4}$ gals. profit, 21 10 0

3683 $\frac{1}{4}$ gals. : 1 gal. :: £272 18 6
or 14733 qrts. : 4 qrts. :: 65502d. : 17 $\frac{3}{4}$ d., or
1s. 5 $\frac{3}{4}$ d. and 1989 over.

50. 1 a. : 17 a. 2 r. 12 p. :: £2 15 0 : £48 6 7 $\frac{1}{2}$
1 a. : 25 a. 1 r. 4 p. :: 17s. 6d. : 22 2 3 $\frac{3}{4}$
£70 8 11 $\frac{1}{4}$

from £96 12 6
take 70 8 11 $\frac{1}{4}$

£26 3 6 $\frac{3}{4}$

then 5s. 4d. : £26 3 6 $\frac{3}{4}$:: 1 a.

or 256 far. : 25131 far. :: 1 a. : 98 a. 0 r. 26 $\frac{3}{4}$ p.

51. 1 yard : 68 yards :: 19s. : £64 12 0,

then £75 - £64 12 0 = £10 8 0 the loss.

52. 25 horse : 20 horse :: 18 hours : 14 hours, 24 min.

53. 90° : 53° . 21' :: 100 grades;

or 5400' : 3201' :: 100 grades : 59 grades, 7',
77" and 42 over.

54. 11 $\frac{1}{2}$ d. : £17 10 7 :: £1 : £365 16 6 $\frac{1}{4}$ amount
of rent;

then £2 17 0 : £365 16 6 $\frac{1}{4}$: 1 acre;

or 2736 far. : 351193 far. :: 1 acre : 128 acres,
1 rood, 17 $\frac{1}{2}$ perches.

Exer.

55.

First cost, £120 0 0
 expenses, 2 10 0

122 10 0

half for gain, 61 5 0

£183 15 0 to be amt. of sales.

43 cwt. 3 qrs. 3 lbs. : 1 cwt. :: £183 15 0;
 or 4903 lbs. : 112 lbs. :: 3675s. : 84s. or £4 4.

56. 20s. : £500 :: 35s. : £875.

57. £100 : £315 11 6 :: £5 7 6;
 or 24000*d.* : 75738*d.* :: 1290*d.* : 4070½*d.*, or
 £16 19 2½.

58. £100 : £1763 : £98 17 0;
 or £100 : £1763 :: 1977s. : 34854s. 6*d.*, or
 £1742 16 6, and 12 over.

59. 1 cwt. : 17 cwt. 3 qrs. 14 lbs. :: £1 12 6;
 or 112 lbs. : 2002 lbs. :: 390*d.* :: 6971½*d.*, or
 £29 0 11½,

and 1 acre : 146 acres, 3 roods :: £1 2 0;
 or 4 roods : 587 roods :: 22s. : £161 8 6,
 and £161 8 6 - £29 0 11½ = £132 7 6¾;
 then 1s. 8½*d.* : £132 7 6¾ :: 1 yard;
 or 82 far. : 127083 far. :: 1 yard : 1549 yards,
 3 qrs. and 14 over.

60. 27 in. : 36 in. :: 59½ yards; or dividing the 1st
 and 2nd by 9 we have:—

3 in. : 4 in. :: 59½ yards : 79½ yards.

61. 1 bar. : 72 bar. 13 stone :: 8s. 2*d.*;
 or 14 st. : 1021 st. :: 98*d.* : 7147*d.* or £29 15 7.
 1 bar. : 40 bar. 15 st. :: £1 7 6;
 or 20 stone : 815 stone :: 450*d.* : 13447½*d.*, or
 £56 0 7½.

1 bar. : 29 bar. 15 stone :: 16s. 4*d.*;
 or 16 stone : 479 stone :: 196*d.* : 5867¾*d.*, or
 £24 8 11¾;

then £29 15 7 + £56 0 7½ + £24 8 11¾ =
 £110 5 2¼. *Answ.*

Exer.

62. From 17th April to 3rd March=321 days;
 then 365 days : 321 days :: £8 8 0 : £7 7 8 $\frac{1}{4}$
 and 329 over.
63. 28 in. : 35 in. :: 56 yards : 70 yards.
64. 39 gals. 3 qrts.
 9
 5 gals. : 357 gals. 3 qrts. :: £3 11 8;
 or 20 qrts. : 1431 qrts. :: 860d. : 61533d., or
 £256 7 9.
65. 10^s : 8^s :: 5^s : 4^s;
 or 1000 : 512 :: 125 : 64.
66. 1728 $\frac{1}{2}$: 512 $\frac{1}{2}$:: 729 $\frac{1}{2}$: 216 $\frac{1}{2}$ or 6.
67. 1° : 360° :: 42 miles : 15120 miles.
 23 hrs. 56 min. 4 sec. : 1 hr. :: 15120 miles;
 or 86164 sec. : 3600 sec. :: 15120 miles : 631 $\frac{3}{4}$
 miles nearly, Dublin.
 1° : 360° :: 42 $\frac{1}{2}$ miles : 15300 miles;
 and 86164 sec. : 3600 sec. :: 15300 miles : 639 $\frac{3}{4}$
 miles, Cork.
 1° : 360° :: 39 $\frac{1}{2}$ miles : 14220 miles;
 and 86164 sec. : 3600 sec. :: 14220 miles : 594
 miles, Belfast.
68. 1 day : 96 days :: 2 $\frac{1}{2}$ lbs. : 240 lbs.
 1 day : 16 days :: 2 $\frac{1}{2}$ lbs. : 40 lbs.;
 then 240 lbs.—40 lbs.=200 lbs.
 96 days : 1 day :: 200 lbs. : 2 lbs. 1 $\frac{1}{3}$ oz of bread.
 1 day : 96 days :: 1 $\frac{1}{4}$ lbs. : 120 lbs.
 1 day : 16 days :: 1 $\frac{1}{4}$ lbs. : 20 lbs.;
 then 120 lbs.—20 lbs.=100 lbs.
 96 days : 1 day :: 100 lbs. : 1 lb. 0 $\frac{2}{3}$ oz. of beef.
69. First cost, £50 15 0
 intended gain, 12 10 0
 total cost, £63 5 0
 3 cwt. 3 qrs. ; 1 lb. :: £63 5 0
 or 420 lbs. : 1 lb. :: 1265s. : 3s. and 5 over.
70. 4 : 3 :: 24 : 18 years.

Exer.

71. 2500000 : 100 :: 212453 : 8 and 20453 over.
 72. £346 17 6 : £100 :: £18 11 5;
 or 83250*d.* : 24000*d.* :: 44570*d.* : £5 7 1 nearly.
 73. 66 feet : $1\frac{1}{2}$ feet :: 1584 yards;
 or 132 : 3 :: 1584 yards : 36 yards;
 then $1584 - 36 = 1548$ yards.
 74. Because $\frac{7}{3} = .318$, &c.,
 $\frac{113}{335} = .337$, &c.; therefore the latter, or
 113 : 335, is the greater ratio.
 75. 156 lbs. : 144 lbs. :: 10 ft. : 9 ft. $2\frac{3}{4}$ in. and 12 over.
 76. 42 in. - 6 in. = 36 in. ;
 then 36 in. : 42 in. :: 240 lbs. : 280 lbs.
 77. $100 - 5 = 95$;
 then $95 :: 570 :: 5 : 30$ absent, and $30 + 570 =$
 600 total.
 78. $\frac{78540}{88888} = .78540$ } therefore the latter is the
 $\frac{88888}{88888} = .78547$ } greater ratio.

DIVISION IN RATIOS.

(Arithmetic, page 102).

1. $79 + 21 = 100 : 2500 \text{ c. ft.} :: 79 \text{ c. ft.} : 1975 \text{ c. ft.}$
 $100 : 2500 :: 21 : 525 \text{ cubic feet.}$
2. $£150 + £90 = £240 : £120 :: £150 : £75.$
 $£240 : £120 :: £90 : £45.$
3. $£250 + £144 + £360 = £754$;
 then $£754 : £84 :: £250 : £27 \text{ 17 } 0\frac{1}{4} \text{ \& 302 over.}$
 $:: £144 : £16 \text{ 0 } 10\frac{1}{4} \text{ nearly.}$
 $:: £360 : £40 \text{ 2 } 1\frac{1}{2} \text{ nearly.}$
4. $2 + 3 + 4 + 5 = 14$;
 then $14 : £20 :: 2 : £2\frac{2}{7}$
 $:: 3 : £4\frac{3}{7}$
 $:: 4 : £5\frac{4}{7}$
 $:: 5 : £7\frac{4}{7}.$
6. $10000 : 3966 :: 112 \text{ lbs.} : 44 \text{ lbs. 6 oz. 11 drs. soda,}$
 $112 \text{ lbs.} - 44 \text{ lbs. 6 oz. 11 drs.} = 67 \text{ lbs. 9 oz. 5 drs.}$
 chlorine.

Exer.

5. 1 a. : 57 a. 2 r. 16 p. :: £1 12 6 : £93 12 0.
 $72 + 80 + 96 + 112 = 360$ sheep;
then 360 : 72 :: £9³ 12 0 : £18 14 4 $\frac{1}{2}$ +
: 80 :: : £20 16 0 +
: 96 :: : £24 19 2 $\frac{1}{2}$ +
: 112 :: : £29 2. 4 $\frac{1}{2}$ +
7. $8 + 4 = 12 : 4 :: £154 : £51 6$ 8 K's part.
: 8 :: : £102 13 4 L's part.
8. £6000 - £1500 = £4500;
then £6000 : £3500 :: £4500 : £2625 A's.
: £2500 :: : £1875 B's.
9. $306 + £252 + £522 = £1080$;
then £1080 : £720 :: £306 : £204.
: £252 : £168.
: £522 : £348.
and £1080 : £720 :: £1 : £0 13 4.
10. $60 + 90 + 150 + 225 = 525$;
then 525 : 35 :: 60 : 4.
: 90 : 6.
: 150 : 10.
: 225 : 15.
11. £280 - £180 A's loss = £100 B's loss.
£100 + £180 = £280;
then £280 : £1680 :: £180 : £1080 A's stock.
: £100 : £600 B' stock.
12. 56 a. 3 r. 16 p. + 85 a. 1 r. 4 p. + 127 a. 2 r. 26 p. =
269 a. 3 r. 6 p.;
then 269 a. 3 r. 6 p. : 138 a. 2 r. :: 56 a. 3 r. 16 p.
or 43166 p. : 22160 p. :: 9096 p. : 29 a. 0 r. 29 $\frac{1}{2}$ p. +
: 13644 p. : 48 a. 3 r. 4 $\frac{1}{2}$ p. -
: 20426 p. : 65 a. 2 r. 6 p. +
13. 1 bl. : 156 bls. :: £1 18 6 : £300 6 0.
1 cwt. : 37 cwt. 3 qrs. 14 lbs. :: £2 6 8 : £88 7 6
1 cwt. : 15 cwt. 2 qrs. 13 lbs. $\times 5$:: £2 16 0 :
£218 12 6;
then £300 6 0 + £88 7 6 + £218 12 6 =
£607 6 0;

PARTNERSHIP WITH TIME.

Exer.

and £607. 6 0 : £200 :: £300 6. 0 :

or 12146s. : 4000s. :: 72072d. : £98 17 11½—

$$:: 21210d. : £29 \quad 2 \quad 1 +$$

∴ 52470d, : £71 19 11½ +

14. $15 + 9 = 24 : 80 \text{ lbs.} :: 15 : 50 \text{ lbs.}$

:: 9 : 30 lbs.

15. $360 : £252 :: £120 \quad 0 \quad 0 : £84 \quad 0 \quad 0$

∴ £72 . 0 0 : £50 8 0

$$:: \text{£}60 \quad 0 \quad 0 : \text{£}42 \quad 0 \quad 0$$

2: £108 0 0 : £75 12 0

16. £180 : £150 :: £200 : £166 13 4 J's stock.

£180 : £240 :: £200 : £266 13 4 L's stock.

17. $12 \text{ mo.} + 15 \text{ mo.} + 18 \text{ mo.} = 45 \text{ mo.}$

45 mo. : 12 mo. :: £120 : £32 D's gain.

: 15 mo. :: £120 : £40 E's ..

: 18 mo. :: £120 : £48 F's ..

PARTNERSHIP WITH TIME.

(Arithmetic, page 104.)

1. C's, £84×15=1260

$$\text{D's, } £108 \times 9 = 972$$
$$2232 : £36 :: 1260 : £20 \quad 6 \quad 5\frac{1}{2}$$

$\therefore 972 : £15\ 13\ 6\frac{1}{3} +$

2. $45 \times 6 = 270$

$$72 \times 4 = 288$$
$$\overline{558} : \pounds 52 :: 270 : \pounds 25 \quad 3 \quad 2\frac{1}{2}$$

∴ 288: £26 16 9½

3. £560 × 35m. = 19600

$$£320 \times 39m. = 12480$$
$$\overline{32080}, £120 :: 19600 : £73 \quad 6 \quad 4+$$

∴ 12480 : £46.13 8-

4. D. $150 \times 49 \text{ mls.} = 7350$

C. $240 \times 21 \text{ mls.} = 5040$

E. $90 \times 56 \text{ mls.} = 5040$

17430 : £96 : 7350 : £40 '9 7½—

each of the others half the remainder.

Exer.

5. R's £800 × 9 mo. = 7200

V's £720 × 5 mo. = 3600

X's £960 × 10 mo. = 9600

20400

20400 : £1728 :: 7200 : £609 17 7½+, R's.

:: 3600 : £304 18 9½+, V's.

:: 9600 : £813 3 6½-, X's.

6. A's £24 16 8 × 6 mo. = 149

B's £13 2 6 × 8 mo. = 105

C's £18 6 8 × 9 mo. = 165

419

419 : £216½ :: 149 : £76 19 9½-, A's.

:: 105 : £54 5 1+, B's.

:: 165 : £85 5 1½+, C's.

7. R's £150 × 3 = 450

S's £175 × 3½ = 612½

T's £300 × 2½ = 750

1812½

1812½ : £1800 :: 450 : £446 17 11½+, R's.

:: 612½ : £608 5 6+, S's.

:: 750 : £744 16 6½+, T's.

COMPOUND PROPORTION.

(Arithmetic, page 106.)

1. 36 cwt. : 42 cwt. } :: £6 : £4 4 0;
25 miles : 15 miles. }

or by dividing 6 and 5:—

6 : 7 } :: £6 : £4 4 0.
5 : 3 }

2. 54 horses : 81 horses } :: 21 weeks : 24½ weeks.
18 cwt : 32 cwt. }

3. 120 mls. : 360 mls. } :: 3 days : 12 days;
9 hrs. : 12 hrs. }

or by dividing 120 and 3:—

1 mile : 3 miles } :: 3 days : 12 days.
3 hours : 4 hours }

Exer.

4. £100 : £775 } :: £4 : £108 10 0
 1 year : 3½ years }
5. 9 persons : 15 persons } :: £150;
 7 mo. : 28 mo. }
- or dividing by 3 and 7:—
- 3 persons : 5 persons } :: £150 : £1000
 1 mo. : 4 mo. }
6. 42 lbs. : 35 lbs. } :: £1 16 0 : £2 10 0
 3 lbs. : 5 lbs. }
7. 36a. 2rs. : 54a. 3r. } :: £45 12 6;
 5 : 6 }
- or 146r. : 219r. } :: 10950d. : £82 2 6
 5 : 6 }
8. £560 : £320 } :: £36;
 15 mo. : 21 mo. }
- or dividing by 80 and 3:—
- 7 : 4 } :: £36 : £28 16 0
 5 : 7 }
9. 35s. : 30s. } :: 5d.
 22 oz. : 108×18oz } :: 5d.
 or 35s. : 30s. } :: 5d.
 32 oz. : 1944 oz. }
- or dividing by 5 and 8:—
- 7 : 6 } :: 5d. : 260½d. =
 4 : 243 } £1 1 8½
10. 72 yards : 84 yards } :: 36 lbs.
 36 inches : 45 inches }
- or dividing by 12 and 9:—
- 6 : 7 } :: 36 lbs. : 52½ lbs. =
 4 : 5 } 3 st. 4½ lbs.
11. £240 : £810 } :: £15;
 21 mo. : 45 mo. }
- 9 : 8 }
- or dividing by 30 and 3:—
- 8 : 27 } :: £15 : £96 8 6¼+
 7 : 15 }
 9 : 8 }

Exer.

$$\left. \begin{array}{l} 12. \ 720 \text{ feet} : 1080 \text{ feet} \\ \quad 24 \text{ feet} : 33 \text{ feet} \\ \quad 12 \text{ feet} : 9 \text{ feet} \\ \quad 12 \text{ hs.} : 10 \text{ hs.} \\ \quad 1 : 3 \end{array} \right\} :: 56 \text{ men.}$$

or dividing by 360, 3, 3, and 2:—

$$\left. \begin{array}{l} 2 \text{ feet} : 3 \text{ feet} \\ 8 \text{ feet} : 11 \text{ feet} \\ 4 \text{ feet} : 3 \text{ feet} \\ 6 \text{ hs.} : 5 \text{ hs.} \\ 1 : 3 \end{array} \right\} :: 56 \text{ men} : 216\frac{2}{3} \text{ men.}$$

INTEREST.

(Arithmetic, page 110.)

Exer.

$$\begin{array}{r} 1. \quad \text{£}240 \quad 4 \quad 4 \text{ at } 5 \\ \quad \quad \quad 5 \\ 100 \overline{)1201 \quad 1 \quad 8} \\ \underline{12 \quad 0 \quad 2\frac{1}{2}+} \end{array}$$

$$\begin{array}{r} 2. \quad \text{£}321 \quad 13 \quad 4 \text{ at } 3 \\ \quad \quad \quad 3 \\ 100 \overline{)965 \quad 0 \quad 0} \\ \underline{9 \quad 13 \quad 0} \end{array}$$

$$\begin{array}{r} 3. \quad \text{£}230 \quad 8 \quad 10\frac{1}{2} \text{ at } 4 \\ \quad \quad \quad 4 \\ 100 \overline{)921 \quad 15 \quad 6} \\ \underline{9 \quad 4 \quad 4\frac{1}{2}+} \end{array}$$

$$\begin{array}{r} 4. \quad \text{£}362 \quad 10 \quad 0 \text{ at } 2 \\ \quad \quad \quad 2 \\ 100 \overline{)725 \quad 0 \quad 0} \\ \underline{7 \quad 5 \quad 0} \end{array}$$

Exer.

$$\begin{array}{r} 5. \quad \text{£}284 \quad 3 \quad 4 \text{ at } 6 \\ \quad \quad \quad 6 \\ 100 \overline{)1705 \quad 0 \quad 0} \\ \underline{17 \quad 1 \quad 0} \end{array}$$

$$\begin{array}{r} 6. \quad \text{£}480 \quad 8 \quad 8 \text{ at } 2\frac{1}{2} \\ \quad \quad \quad 2\frac{1}{2} \\ \underline{960 \quad 17 \quad 4} \\ 240 \quad 4 \quad 4 \end{array}$$

$$\begin{array}{r} 100 \overline{)1201 \quad 1 \quad 8} \\ \underline{12 \quad 0 \quad 2\frac{1}{2}+} \end{array}$$

$$\begin{array}{r} 7. \quad \text{£}220 \quad 6 \quad 8 \text{ at } 2\frac{3}{4} \\ \quad \quad \quad 3 \\ \underline{661 \quad 0 \quad 0} \\ \frac{1}{4} \text{ off, } 55 \quad 1 \quad 8 \\ 100 \overline{)605 \quad 18 \quad 4} \\ \underline{6 \quad 1 \quad 2+} \end{array}$$

Exer.

8. £136 3 10½ at 3½

$$\begin{array}{r}
 3\frac{1}{2} \\
 408 \text{ 11 } 7\frac{1}{2} \\
 68 \text{ 1 } 11\frac{1}{2} \\
 100) 476 \text{ 13 } 6\frac{1}{2} \\
 \underline{4 \text{ 15 } 4+}
 \end{array}$$

9. £160 16 8 at 1½

$$\begin{array}{r}
 1\frac{1}{2} \\
 160 \text{ 16 } 8 \\
 80 \text{ 8 } 4 \\
 100) 241 \text{ 5 } 0 \\
 \underline{2 \text{ 8 } 3}
 \end{array}$$

10. £566 13 4 at 5

$$\begin{array}{r}
 5 \\
 100) 2833 \text{ 6 } 8 \\
 \underline{28 \text{ 6 } 8}
 \end{array}$$

11. £335 19 9 at 4½

$$\begin{array}{r}
 4\frac{1}{2} \\
 1343 \text{ 19 } 0 \\
 83 \text{ 19 } 11\frac{1}{2} \\
 100) 1427 \text{ 18 } 11\frac{1}{2} \\
 \underline{14 \text{ 5 } 7+} \\
 335 \text{ 19 } 9+ \\
 \underline{350 \text{ 5 } 4+}
 \end{array}$$

12. £325 2 6 at 2½

$$\begin{array}{r}
 2\frac{1}{2} \\
 650 \text{ 5 } 0 \\
 54 \text{ 3 } 9 \\
 100) 704 \text{ 8 } 9 \\
 \underline{7 \text{ 0 } 10\frac{1}{2}+} \\
 325 \text{ 2 } 6 \\
 \underline{332 \text{ 3 } 4\frac{1}{2}+}
 \end{array}$$

Exer.

13. £162 11 3¼ at 3½

$$\begin{array}{r}
 3\frac{1}{2} \\
 487 \text{ 13 } 9\frac{1}{2} \\
 60 \text{ 19 } 2\frac{1}{2} \\
 100) 548 \text{ 13 } 0\frac{1}{2} \\
 \underline{5 \text{ 9 } 8\frac{1}{2}} \\
 162 \text{ 11 } 3\frac{1}{2} \\
 \underline{168 \text{ 1 } 0}
 \end{array}$$

14. £256 4 10½ at 4½

$$\begin{array}{r}
 4\frac{1}{2} \\
 1024 \text{ 19 } 6 \\
 85 \text{ 8 } 3\frac{1}{2} \\
 100) 1110 \text{ 7 } 9\frac{1}{2} \\
 \underline{11 \text{ 2 } 0\frac{1}{2}} \\
 256 \text{ 4 } 10\frac{1}{2} \\
 \underline{267 \text{ 6 } 11\frac{1}{2}}
 \end{array}$$

15. £568 6 8 at 3

$$\begin{array}{r}
 3 \\
 100) 1705 \text{ 0 } 0 \\
 \underline{17 \text{ 1 } 0} \\
 568 \text{ 6 } 8 \\
 \underline{585 \text{ 7 } 8}
 \end{array}$$

16. £832 0 0 at 2½

$$\begin{array}{r}
 2\frac{1}{2} \\
 1664 \text{ 0 } 0 \\
 416 \text{ 0 } 0 \\
 100) 2080 \text{ 0 } 0 \\
 \underline{20 \text{ 16 } 0} \\
 832 \text{ 0 } 0 \\
 \underline{852 \text{ 16 } 0}
 \end{array}$$

Exer.

24. £87 7 4½ at 6

$$\begin{array}{r} 100 \overline{) 524 \ 4 \ 1\frac{1}{2}} \\ 5 \ 4 \ 10 \end{array}$$

365 : 351 :: £5 4 10 :
£5 0 9¾.

25. £483 17 4 at 5

$$\begin{array}{r} 100 \overline{) 2419 \ 6 \ 8} \\ 24 \ 3 \ 10\frac{1}{2} \\ 6 \\ \hline 145 \ 3 \ 3 \end{array}$$

12 mo. : 9 mo. ::
£24 3 10½ : £18 2 10¾;
and £145 3 3 +
£18 2 10¾ = £163 6 1¾;
then £483 17 4 +
£163 6 1¾ = £647 3 5+.

26. £196 9 5½ at 3½

$$\begin{array}{r} 589 \ 8 \ 4\frac{1}{2} \\ 98 \ 4 \ 8\frac{3}{4} \\ 100 \overline{) 687 \ 13 \ 1\frac{1}{2}} \\ 6 \ 17 \ 6\frac{1}{4} \end{array}$$

12 mo. : 29 mo. ::
£6 17 6¼ : £16 12 4¼;
then £196 9 5½ +
£16 12 4¼ = £213 1 9¾.

Exer.

27. £24 14 9¼ at 4

$$\begin{array}{r} 100 \overline{) 98 \ 19 \ 1} \\ 0 \ 19 \ 9\frac{1}{2} \\ 4 \\ \hline 3 \ 19 \ 2 \end{array}$$

365 : 187 :: £0 19 9½ :
£0 10 1½
then £24 14 9¼ +
£3 19 2 + £0 10 1½ =
£29 4 0¾.

28. £16 13 6¾ at 6

$$\begin{array}{r} 100 \overline{) 100 \ 1 \ 4\frac{1}{2}} \\ 1 \ 0 \ 0+ \end{array}$$

365 : 215 :: £1 0 0 :
£0 11 9¼;
then £16 13 6¾ +
£0 11 9¼ = £17 5 4.

29. £314 11 8 at 5

$$\begin{array}{r} 100 \overline{) 1572 \ 18 \ 4} \\ 15 \ 14 \ 7 \end{array}$$

365 : 186 :: £15 14 7 :
£8 0 3½+;
then £314 11 8 +
£8 0 3½ = £322 11 11½+

Exer.

30. £78 3 10 at 3½

$$\begin{array}{r} 3\frac{1}{2} \\ \hline 234 \ 11 \ 6 \\ 39 \ 1 \ 11 \\ \hline 100)273 \ 13 \ 5 \\ \hline 2 \ 14 \ 8\frac{1}{2} \end{array}$$
$$365 : 486 :: £2 \ 14 \ 8\frac{1}{2} :$$

£3 12 10½;

then £78 3 10 +

$$\text{£}3\ 12\ 10\frac{1}{4} = \text{£}81\ 16\ 8\frac{1}{4}$$

31. £516 14 8½ at 4

$$\begin{array}{r} 100 \overline{) 2066 \ 18 \ 10} \\ \underline{20 \ 13 \ 41} \end{array}$$
$$365 : 147 :: £20 \ 13 \ 4\frac{1}{2} :$$

£8 6 5³;

then £516 14 8½ +

$\pounds 8\ 6\ 5\frac{3}{4} = \pounds 525\ 1\ 2\frac{1}{4}$.

32. £98 7 5 at 2½

$$\begin{array}{r} 2\frac{3}{4} \\ \hline 196 \quad 14 \quad 10 \\ 49 \quad 3 \quad 8\frac{1}{8} \\ 24 \quad 11 \quad 10\frac{1}{4} \\ \hline 100(270 \quad 10 \quad 4\frac{3}{4} \\ \hline 2 \quad 14 \quad 1\frac{1}{2} \end{array}$$
$$365 : 182 :: \text{£}2 \ 14 \ 1\frac{1}{4} :$$

£1 6 11½ +;

then £98 7 5 +

$$\pounds 1\ 6\ 11\frac{1}{2} = \pounds 99\ 14\ 4\frac{1}{2} +$$

Exer.

33. £516 11 8 at 5 for

60 days:—

$$\begin{array}{r} \text{£}516 \text{ } 11 \text{ } 8 \\ \quad \quad 10 \\ \hline 5165 \text{ } 16 \text{ } 8 \\ \quad \quad 10 \\ \hline 51658 \text{ } 6 \text{ } 8 \\ \quad \quad 6 \\ \hline 309950 \text{ } 0 \text{ } 0 \\ \hline 4 \text{ } 4 \text{ } 11 \end{array}$$

34. £94 3 4 at 5 for

220 days :—

$$\begin{array}{r}
 \text{£}94 \quad 3 \quad 5 \\
 8 \\
 \hline
 753 \quad 7 \quad 4 \\
 10 \\
 \hline
 7533 \quad 13 \quad 4 \\
 11 \\
 \hline
 82870 \quad 6 \quad 8 \\
 2 \\
 \hline
 73000)165740 \quad 13 \quad 4 \\
 2 \quad 5 \quad 4 \quad +
 \end{array}$$

35. £723 16 0 at 3 for

212 days:—

$$\begin{array}{r}
 \text{£}723 \ 10 \ 0 \\
 6 \\
 \hline
 4342 \ 16 \ 0 \times 12 \\
 10 \\
 \hline
 43428 \ 0 \ 0 \\
 2 \\
 \hline
 868560 \ 0 \ 0 \\
 52113 \ 12 \ 0 \\
 73000)920673 \ 12 \ 0 \\
 \hline
 12 \ 12 \ 2\frac{1}{2}
 \end{array}$$

Exer.

36. From Feb. 11-44 to
Nov. 29-45 are 657 days;
 $657 = 9 \times 73$ and $73000 =$
 1000×73 , therefore we
may use 9 as multiplier and
1000 as divisor:—

$$\begin{array}{r}
 \text{£}96 \ 17 \ 6 \text{ at } 6 \\
 \quad \quad \quad 12 \\
 \hline
 1162 \ 10 \ 0 \\
 \quad \quad \quad 9 \\
 \hline
 1000) 10462 \ 10 \ 0 \\
 \hline
 10 \ 9 \ 3
 \end{array}$$

37.

£413 9 10 at $2\frac{1}{2}$ for 177 days,

$$\begin{array}{r}
 \quad \quad \quad 5 \\
 \hline
 2067 \ 9 \ 2 \times 1 \quad 177 = 8 \times 11 \times 2 + 1 \\
 \quad \quad \quad 8 \\
 \hline
 16539 \ 13 \ 4 \\
 \quad \quad \quad 11 \\
 \hline
 181936 \ 6 \ 8 \\
 \quad \quad \quad 2 \\
 \hline
 363872 \ 13 \ 4 \\
 \quad \quad \quad 2067 \ 9 \ 2 \\
 \hline
 73000) 365940 \ 2 \ 6 \\
 \hline
 \quad \quad \quad 5 \ 0 \ 3
 \end{array}$$

38.

£52 10 8 at $4\frac{1}{4}$ for 190 days,

$$\begin{array}{r}
 \quad \quad \quad 8\frac{1}{2} \\
 \hline
 420 \ 5 \ 4 \quad 190 = 3 \times 9 \times 7 + 1 \\
 \quad \quad \quad 26 \ 5 \ 4 \\
 \hline
 446 \ 10 \ 8 \times 1 \\
 \quad \quad \quad 3 \\
 \hline
 1339 \ 12 \ 0 \\
 \quad \quad \quad 9 \\
 \hline
 12056 \ 8 \ 0 \\
 \quad \quad \quad 7 \\
 \hline
 84394 \ 16 \ 0 \\
 \quad \quad \quad 446 \ 10 \ 8 \\
 \hline
 73000) 84841 \ 6 \ 8 \\
 \hline
 \quad \quad \quad 1 \ 3 \ 2\frac{1}{4}
 \end{array}$$

Exer.

39.

$$\begin{array}{r} \text{£}500 \quad 0 \quad 0 \text{ at } 6 \\ \hline \end{array}$$

$$100 \overline{)3000} \quad 0 \quad 0$$

$$12 \text{ mo.} : 7 \text{ mo.} :: 30 \quad 0 \quad 0 : \text{£}17 \quad 10 \quad 0$$

$$\begin{array}{r} \text{£}500 - \text{£}110 = \text{£}390 \quad 0 \quad 0 \\ \hline \end{array}$$

$$100 \overline{)2340} \quad 0 \quad 0$$

$$12 \text{ mo.} : 3 \text{ mo.} :: 23 \quad 8 \quad 0 : \quad 5 \quad 17 \quad 0$$

$$\begin{array}{r} \text{£}390 - \text{£}80 = \text{£}310 \quad 0 \quad 0 \\ \hline \end{array}$$

$$100 \overline{)1860} \quad 0 \quad 0$$

$$12 \text{ mo.} : 11 \text{ mo.} :: 18 \quad 12 \quad 0 : \quad 17 \quad 1 \quad 0$$

$$\begin{array}{r} \text{£}310 - \text{£}270 = \text{£}40 \quad 0 \quad 0 \\ \hline \end{array}$$

$$100 \overline{)240} \quad 0 \quad 0$$

$$12 \text{ mo.} : 5 \text{ mo.} :: \quad 2 \quad 8 \quad 0 : \quad 1 \quad 0 \quad 0$$

$$\text{Answ. } \text{£}41 \quad 8 \quad 0$$

or thus :—

$$\text{£}500 \times 7 = 3500$$

$$\underline{110}$$

$$390 \times 3 = 1170$$

$$\underline{80}$$

$$310 \times 11 = 3410$$

$$\underline{270}$$

$$40 \times 5 = 200$$

$$1200 \quad : \quad 8280 : \text{£}6 :: \text{£}41 \quad 8 \quad 0$$

$$\begin{array}{l} 41. \text{ £}4 \quad : \text{£}33 \quad 5 \quad 10 \\ \quad 17 \text{ mo.} \quad : 12 \text{ mo.} \\ \quad \text{or } 960d. : 7990d. \\ \quad 17 \text{ mo.} \quad : 12 \text{ mo.} \end{array} \left. \vphantom{\begin{array}{l} 41. \text{ £}4 \quad : \text{£}33 \quad 5 \quad 10 \\ \quad 17 \text{ mo.} \quad : 12 \text{ mo.} \\ \quad \text{or } 960d. : 7990d. \\ \quad 17 \text{ mo.} \quad : 12 \text{ mo.} \end{array}} \right\} :: \text{£}100;$$

$$\left. \vphantom{\begin{array}{l} 41. \text{ £}4 \quad : \text{£}33 \quad 5 \quad 10 \\ \quad 17 \text{ mo.} \quad : 12 \text{ mo.} \\ \quad \text{or } 960d. : 7990d. \\ \quad 17 \text{ mo.} \quad : 12 \text{ mo.} \end{array}} \right\} :: \text{£}100 : \text{£}587 \quad 10 \quad 0$$

Exer.

40. $\text{£}1800 \times 83 \text{ days} = 149400$

486

$1314 \times 59 = 77526$

390

$924 \times 117 = 108108$

618

$306 \times 23 = 7038$

217

$89 \times 143 = 12727$

$36500 : 354799 :: \text{£}4 : \text{£}38 \ 17 \ 7$

42. $\text{£}3\frac{1}{2} : \text{£}5$ } :: $\text{£}100$
 545 ds. : 365 ds. }
 or 7 : 10 } :: $\text{£}100 : \text{£}95 \ 13 \ 6$
 345 ds. : 365 ds. }
43. $\text{£}4 : \text{£}65$ } :: 1 yr. : 2 yrs. 1 mo.
 $\text{£}780 : \text{£}100$ }
44. $\text{£}5 : \text{£}124$ } :: 1 yr. : 2 yrs. $175\frac{1}{2}$ dys.
 $\text{£}1000 : \text{£}100$ }
 which, from Jan., 1841, will fall on June 26, 1843.
45. $\text{£}240 : \text{£}100$ } :: $\text{£}36 : \text{£}3\frac{1}{4}$.
 4 yrs. : 1 yr. }
46. $\text{£}205 : \text{£}100$ } :: $\text{£}13 : \text{£}10 \ 17 \ 5+17$.
 7 mo. : 12 mo. }
47. 1 year : 3 years :: $\text{£}5 : \text{£}15$.
 $\text{£}115 : \text{£}276 :: \text{£}100 : \text{£}240$.
48. 12 mo. : 9 mo. :: $\text{£}4 : \text{£}3$,
 $\text{£}103 : \text{£}468 :: \text{£}100 : \text{£}454 \ 7 \ 4\frac{2}{3}\%$.

COMPOUND INTEREST.

(Arithmetic, page 119.)

	£	£	s.	d.	£	£	s.	d.
1.	100	: 60	0	0	:: 4	: 2	8	0
	—	: 62	8	0	:: 4	: 2	9	11
	—	: 64	17	11	:: 4	: 2	11	11
	—	: 67	9	10	:: 4	: 2	13	$11\frac{1}{2}$
	—	: 70	3	$9\frac{1}{2}$:: 4	: 2	16	$1\frac{1}{2}$
								$\text{£}12 \ 19 \ 11\frac{1}{2}+$

Exer.

	£	£	s.	d.	£	£	s.	d.
2.	100	: 150	0	0	:: 2 : 3	0	0	
	—	: 153	0	0	:: 2 : 3	1	2½	
	—	: 156	1	2½	:: 2 : 3	2	5	
	—	: 159	3	7½	:: 2 : 3	3	8	
	—	: 162	7	3½	:: 2 : 3	4	11½	
	—	: 165	12	2½	:: 2 : 3	6	3	
	—	: 168	18	5½	:: 2 : 3	7	6¾	
	—	: 172	6	0½	:: 2 : 3	8	11	
	—	: 175	14	11½	:: 2 : 3	10	3½	
	and 179	5	2¾	+	= the amount.			

	£	£	s.	d.	£	£	s.	d.
3.	100	: 300	0	0	:: 3 : 9	0	0	
	—	: 309	0	0	:: 3 : 9	5	4¾	
	—	: 318	5	4¾	:: 3 : 9	10	11½	
	—	: 327	16	4¼	:: 3 : 9	16	8¼	
	—	: 337	13	0½	:: 3 : 10	2	7	
	—	: 347	15	7½	:: 3 : 10	8	8	
	—	: 358	4	3½	:: 3 : 10	14	11	
	and 368	19	2½	+	= the amount.			

	£	£	s.	d.	£	£	s.	d.
4.	100	: 1000	0	0	:: 3 : 30	0	0	
	—	: 1030	0	0	:: 3 : 30	18	0	
	—	: 1060	18	0	:: 3 : 31	16	6½	
	—	: 1092	14	6½	:: 3 : 32	15	7½	
	—	: 1125	10	2	:: 3 : 33	15	3½	
	—	: 1159	5	5½	:: 3 : 34	15	6¾	
						£194	1	0¼+

COMMISSION, BROKERAGE, &c. &c.

(Arithmetic, page 120.)

Exer.

1.	£537	16	8 at 2½
			2½
	1075	13	4
	268	18	4
100)	1344	11	8
	13	8	11

Exer.

2.	£382	10	0 at 1¼
			1¼
	382	10	0
	95	12	6
100)	478	2	6
	4	15	7½

Exer.

3. £2968 13 8½ at 1½

$$\begin{array}{r}
 2968 \ 13 \ 8\frac{1}{2} \\
 \underline{1484 \ 6 \ 10\frac{1}{2}} \\
 100)4453 \ 0 \ 6\frac{3}{4} \\
 \underline{44 \ 10 \ 7\frac{1}{4}}
 \end{array}$$

$$\begin{array}{r}
 4. 100)£344 \ 17 \ 6 \\
 \underline{3 \ 8 \ 11\frac{1}{2}} \\
 341 \ 8 \ 6\frac{1}{2}
 \end{array}$$

5. £1953 15 0 at 2s. 6d.

$$\begin{array}{r}
 1953 \ 15 \ 0 \\
 \underline{1 \ 8} \\
 100)244 \ 4 \ 4\frac{1}{2} \\
 \underline{2 \ 8 \ 10}
 \end{array}$$

Exer.

6. £205 15 0 at 5

$$\begin{array}{r}
 205 \ 15 \ 0 \\
 \underline{5} \\
 100)1028 \ 15 \ 0 \\
 \underline{10 \ 5 \ 9}
 \end{array}$$

7. £1234 0 0 at 6s. 8d.

$$\begin{array}{r}
 1234 \ 0 \ 0 \\
 \underline{1 \ 3} \\
 100)411 \ 6 \ 8 \\
 \underline{4 \ 2 \ 3}
 \end{array}$$

8.	cwt.	:	cwt.	qrs.	lbs.	:	£	s.	d.	:	£	s.	d.
	1	:	413	2	8	::	2	6	0	:	951	4	3½
	1	:	73	0	16	::	2	17	4	:	209	13	4

$$\text{purchase} = £1160 \ 17 \ 7\frac{1}{2}$$

cwt.	:	cwt.	qrs.	lbs.	:	£	s.	d.	:	£	s.	d.
1	:	413	2	8	::	2	18	4	:	1206	5	0
1	:	73	0	16	::	3	14	0	:	270	12	7

$$\text{sales} = £1476 \ 17 \ 7$$

$$£1476 \ 17 \ 7 \times 2\frac{1}{2} \div 100 = £36 \ 18 \ 5\frac{1}{4} \text{ factorage.}$$

$$\text{Carriage, cooperage, \&c.} \quad 8 \ 0 \ 10$$

$$\text{Factorage and charges} = £44 \ 19 \ 3\frac{1}{4}$$

$$\text{Purchase, ...} \quad \dots \quad £1160 \ 17 \ 7\frac{1}{2}$$

$$\text{Factorage, \&c.} \quad \dots \quad 44 \ 19 \ 3\frac{1}{4}$$

$$\text{Prime cost, ...} \quad \dots = £1205 \ 16 \ 10\frac{3}{4}$$

Exer.

From sales, £1476 17 7
 Take factorage, &c., 44 19 3½

Nett proceeds, ...=£1431 18 3½

From nett proceeds, £1431 18 3½

Take prime cost, ... 1205 16 10½

Gain, = £226 1 5

9. £100

6

£94 : £799 :: £100 : £850.

10. £1500+£3108=£4608

£100

4

£96 : £4608 :: £100 : £4800.

11. 7767401-6801827=965574

6801827 : 965574 :: 100 : 14 +

8205382-7767401=436981

7767401 : 436981 :: 100 : 5½ +.

12. 1367563 : 319792 :: 100 : 23½

13. 100 : 2983 :: 27 : 805 +.

14. £1164046 : £360 18s. 8d. :: £100 ;

or £1164046 : 86624d. :: £100 : 7¼d. +.

15. 100 : 105½ :: 182713 ;

or 200 : 211 :: 182713 : 192762 +.

16. 1805688-1599068=206620 ; then

1599068 : 206620 :: 100 : 13 nearly.

2093456-1805688=287768 ; then

1805688 : 287768 :: 100 : 16 nearly.

2365114-2093456=271658 ; then

2093456 : 271658 :: 100 : 13 nearly.

STOCKS.

(Arithmetic, page 125.)

2. £100 : 1000 :: £92½ - ½ :

or 1 : 10 :: 92½ : £925.

3. £75 : £100 :: £675 : £900.

Exer.

$$4. \text{£}72 : \text{£}90 :: \text{£}800 : \text{£}1000.$$

$$5. \text{£}100 : 83\frac{3}{8} \times \frac{1}{8} :: \text{£}2000 ;$$

$$\text{or } \text{£}100 : \text{£}83\frac{1}{2} :: \text{£}2000 : \text{£}1670.$$

$$6. \text{£}175 : \text{£}100 :: \text{£}1400 : \text{£}800.$$

$$7. \text{£}100 : \text{£}9692 \ 9 \ 9\frac{1}{4} :: \text{£}92\frac{1}{8} \times \frac{1}{8} ;$$

$$\text{or } \text{£}100 : \text{£}9692 \ 9 \ 9\frac{1}{4} :: \text{£}92\frac{1}{4} ;$$

$$\text{or } 400 : 9304789 \text{ f.} :: 369 : \text{£}8941 \ 6 \ 5.$$

$$8. \text{£}93 - \text{£}91\frac{1}{2} = \text{£}1\frac{1}{2} ;$$

$$\text{then } \text{£}100 : \text{£}2000 :: \text{£}1\frac{1}{2} : \text{£}30.$$

$$9. \quad \text{£}4000 \ 0 \ 0 \text{ at } 3\frac{1}{2}$$

$$\begin{array}{r} 3\frac{1}{2} \\ 100 \overline{) 14000 \ 0 \ 0} \\ \underline{140 \ 0 \ 0} \end{array} = \text{his income before the transfer.}$$

From 96

take $\frac{1}{8}$

$$100 : 4000 :: 95\frac{1}{8} : \text{£}3835 = \text{the amt. he receives on selling out.}$$

From 95

take $\frac{1}{8}$

$$94\frac{1}{8} : 2000 :: 100 : \text{£}2108 \ 0 \ 8 ;$$

and $\text{£}2108 \ 0 \ 8$ at 3

$$\begin{array}{r} 3 \\ 100 \overline{) 6324 \ 2 \ 0} \\ \underline{63 \ 4 \ 9\frac{3}{4}} \end{array} \text{ in the 3 per cents.}$$

From 92

take $\frac{1}{8}$

$$91\frac{1}{8} : 1835 :: 100 : \text{£}1997 \ 5 \ 5 ;$$

and $\text{£}1997 \ 5 \ 5$ at $3\frac{1}{2}$

$$\begin{array}{r} 3\frac{1}{2} \\ 100 \overline{) 6990 \ 8 \ 11\frac{1}{2}} \\ \underline{69 \ 18 \ 1} \text{ in the } 3\frac{1}{2} \text{ per cents.} \\ \underline{63 \ 4 \ 9\frac{3}{4}} \\ 133 \ 2 \ 10\frac{3}{4} \end{array}$$

$$\text{£}140 - \text{£}133 \ 2 \ 10\frac{3}{4} = \text{£}6 \ 17 \ 1\frac{1}{4} \text{ he loses by the transfer.}$$

DISCOUNT.

Exer.

(Arithmetic, p. 126.)

1. £100

$$\begin{array}{r} 4 \\ \hline \end{array} \quad \begin{array}{ccccccc} \text{£} & \text{s.} & \text{d.} & \text{£} & \text{£} & \text{s.} & \text{d.} \end{array}$$

$$\text{£104} : 239 \ 14 \ 4\frac{1}{2} :: 100 : 230 \ 10 \ 0 \text{ nearly.}$$

2. £100

$$\begin{array}{r} 2 \\ \hline \end{array} \quad \text{£102} : 369 \ 15 \ 0 :: 100 : 362 \ 10 \ 0.$$

3. £100

$$\begin{array}{r} 3\frac{1}{2} \\ \hline \end{array} \quad \text{£103}\frac{1}{2} : 140 \ 19 \ 2\frac{1}{2} :: 100 : 136 \ 3 \ 10\frac{1}{2}.$$

4. £100

$$\begin{array}{r} 2\frac{1}{2} \\ \hline \end{array} \quad \text{£102}\frac{1}{2} : 852 \ 16 \ 0 :: 100 : 832 \ 0 \ 0.$$

5. £100

$$\begin{array}{r} 3\frac{1}{2} \\ \hline \end{array} \quad \text{£103}\frac{1}{2} : 594 \ 15 \ 9\frac{1}{2} :: 100 : 574 \ 13 \ 6\frac{1}{2} \text{ nearly.}$$

6. £100

$$\begin{array}{r} 5 \\ \hline \end{array} \quad \text{£105} : 759 \ 0 \ 0 :: 100 : 722 \ 17 \ 1\frac{1}{2}+$$

7. £100

$$\begin{array}{r} 8 \\ \hline \end{array} \quad \text{£108} : 384 \ 4 \ 2\frac{1}{2} :: 100 : 355 \ 15 \ 0.$$

8. £100

$$\begin{array}{r} 5 \\ \hline \end{array} \quad \text{£105} : 747 \ 18 \ 9\frac{1}{2} :: 100 : 712 \ 6 \ 5\frac{1}{2}+.$$

9. 12 mo. : 7 mo. :: £4 : £2½

$$\text{£102}\frac{1}{2} : \text{£37} \ 12 \ 6\frac{1}{2} :: \text{£100} : \text{£36} \ 15 \ 4+.$$

10.

6 yrs. 9 mo.

12

$$12 \text{ mo.} : \overline{81} \text{ mo.} :: \text{£5} : \text{£33}\frac{1}{2}.$$

$$\text{£133}\frac{1}{2} : \text{£647} \ 3 \ 5 :: \text{£100} : \text{£483} \ 17 \ 3\frac{1}{2}+.$$

Exer.

11. 12 mo. : 9 mo. :: £4 : £3.

£103 : £207 16 1½ : £100 : £201 15 1+.

12. 1 yr. 8 mo.

12

12 mo. : 20 mo. :: £6 : £10.

£110 : £543 4 4 :: £100 : £493 16 8.

13. 365 ds. : 351 ds. :: £6 : £5 15 4¾.

£105 15 4¾ : £92 8 2 :: £100 : £87 7 4+.

14. 365 ds. : 146 ds. :: £3½ : £1 8 0.

£101 8 0 : £736 18 1¾ :: £100 : £726 14 8+.

15. 365 ds. : 60 ds. :: £5 : £0 16 5¼.

£100 16 5¼ : £516 11 8 : £100 : £512 7 5¼+.

16. From July 1, to December 25 = 177 days.

365 ds. : 177 ds. :: £2½ : £1 4 3 nearly.

£101 4 3 : £413 9 10 :: £100 : £408 10 9;

then £413 9 10 - £408 10 9 = £4 19 1.

EQUATION OF PAYMENTS.

(Arithmetic, page 129.)

$$\begin{array}{rcl}
 1. & & £1250 \times 4 = 5000 \\
 & & 1500 \times 12 = 18000 \\
 & & 1800 \times 15 = 27000 \\
 & & \hline
 & & 4550 \qquad \frac{50000}{4550} = 11 \text{ mo. nearly.}
 \end{array}$$

$$\begin{array}{rcl}
 3. & & £120 \times 5 = 600 \\
 & & 125 \times 4 = 500 \\
 & & 500 \times 8 = 4000 \\
 & & \hline
 & & 745 \quad) \quad 5100 (6 \text{ mo. } 25\frac{1}{2} \text{ ds.} \\
 & & \quad 4470 \\
 & & \quad \hline
 & & \quad 630 \\
 & & \quad 30 \\
 & & \quad \hline
 & & \quad 18900 \\
 & & \quad 18625 \\
 & & \quad \hline
 & & \quad 275 \\
 & & \quad \hline
 & & \quad 275 \\
 & & \quad \hline
 & & \quad 745 = \frac{1}{3} \text{ nearly.}
 \end{array}$$

Exer.

$$\begin{array}{r}
 2. \quad \quad \quad \pounds 400 \times 8 = 3200 \\
 \quad \quad \quad \pounds 600 \times 3 = 1800 \\
 \hline
 \quad \quad \quad 1000 \quad) \quad 5000 \text{ (5 mo.} \\
 \quad \quad \quad \quad \quad 5000 \\
 \hline
 \end{array}$$

BARTER.

(Arithmetic, page 129.)

1. 1 yd. : $146\frac{1}{2}$ yds. :: $\pounds 1 \ 5 \ 0$: $\pounds 183 \ 2 \ 6$,
 $\pounds 183 \ 2 \ 6 \div 4 = \pounds 45 \ 15 \ 7\frac{1}{2}$ in cash,
 $\pounds 183 \ 2 \ 6 - \pounds 45 \ 15 \ 7\frac{1}{2} = \pounds 137 \ 6 \ 10\frac{1}{2}$;
then $\pounds 5 \ 15 \ 8\frac{1}{2}$: $\pounds 137 \ 6 \ 10\frac{1}{2}$:: 1 cwt.;
or 2777 halfpence : 65925 halfpence :: 1 cwt. :
23 cwt. 2 qrs. $26\frac{3}{4}$ lbs.
2. 1 yr. : $3\frac{1}{2}$ yrs. :: $\pounds 4$: $\pounds 14$,
 $\pounds 114$: $\pounds 1026$: $\pounds 100$. $\pounds 900$,
 $\pounds 1026 - \pounds 900 = \pounds 126 =$ discount,
 $\pounds 100$: $\pounds 130$ } :: $\pounds 5$: $\pounds 25$ interest,
1 yr. : 4 yrs. }
 $\pounds 126 - \pounds 26 = \pounds 100$ to be made up in tea,
4s. 2d. : $\pounds 100$:: 1 lb. : 480 lbs. = 4 cwt. 1 qr. 4 lb.

PROFIT AND LOSS.

(Arithmetic, page 130.)

1. $\pounds 2 \ 6 \ 8 - \pounds 1 \ 17 \ 4 = \pounds 0 \ 9 \ 4$.
1 cwt : 27 cwt. 1 qr. 19 lbs. :: 9s. 4d,
or 112 lbs. : 3070 lbs. :: 112d. : $\pounds 12 \ 15 \ 10$;
and $\pounds 1 \ 17 \ 4$: $\pounds 100$:: 9s. 4d. : $\pounds 25$ per cent.
2. $14s. \ 7d. - 14s. = 7d.$,
1 ream : 120 reams :: 7d. : $\pounds 3 \ 10 \ 0$;
and $14s. \ 7d.$: $\pounds 100$:: 7d. : $\pounds 4$ per cent.
3. $7s. \ 6d. - 6s. \ 3d. = 1s. \ 3d.$,
 $6s. \ 3d.$: $\pounds 100$:: $1s. \ 3d.$: $\pounds 20$ per cent. on the
whiskey;
 $\pounds 3 \ 10 \ 0 - \pounds 2 \ 16 \ 0 = 14s.$,
 $\pounds 2 \ 16 \ 0$: $\pounds 100$:: 14s. : $\pounds 25$ per cent. on the
tallow;
 $\pounds 25 - \pounds 20 = \pounds 5$ advantage on the tallow per cent.

Exer.

4. £110 : £5 2 8 :: £100 : £4 13 4.
 5. £109 : 6s. 9 $\frac{3}{4}$ d. :: £100 : 6s. 3d. first cost;
 then 6s. 9 $\frac{3}{4}$ d. - 6s. 3d. = 6 $\frac{3}{4}$ d.;
 1 gal. : 156 gals. :: 6 $\frac{3}{4}$ d. : £4 7 9.
 6. £100
 6
 £94 : £4 14 0 : £100 : £5 per quarter,
 £5 - £4 14 0 = 6s.;
 6s. : £25 1 0 :: 1 qr. : 83 $\frac{1}{4}$ quarters,
 1 qr. : 167 qrs. £1 10 0 : £250 10 0,
 £250 10 0 : £25 1 0 :: £100 : £10 per cent.

EXCHANGE.

(Arithmetic, page 138.)

1. £1 : £57 4 0 :: 23f. 15c.;
or 20s. 1144s. :: 2315c. : 1324f. 18c.
2. £1 : £156 7 6 :: 24f. 8c.;
or 240d. : 37530d. :: 2408c. : 3765f. 51c.
3. £1 : £120 10 9 $\frac{1}{4}$:: 23f. 50c.;
or 960 far. :: 115717 far. :: 2350c. : 2832f. 65c.
+ $\frac{5}{8}$.
4. 23f. 52c. : 23066f. 26c. :: £1;
or 2352c. : 2306626c. :: £1 : £980 14 2+.
5. 2404c. : 3275f. 45c. :: £1;
or 24f. 4c. : 327545c. :: £1 : £136 5 0.
6. 23f. 36c. : 19086f. 58c. :: £1;
or 2336c. : 1908658c. :: £1 : £817 1 3.
7. 1p. : 516p. 6r. 32m. :: 36d.;
or 272m. : 140588m. :: 36d. :: £77 10 7+.
8. 1p. : 1823p. 17m. :: 38 $\frac{1}{2}$ d.;
or 272m. : 495873m. :: 38 $\frac{1}{2}$ d. : £292 8 11 $\frac{1}{4}$ +
9. 39d. : £173 11 8 :: 1p.;
or 39d. : 41660d. :: 1p. : 1068p. 1r. 21 $\frac{3}{4}$ m. +.
10. 40d. : £421 17 6 :: 1p.;
or 40d. : 101250d. :: 1p. : 2531p. 2r.
11. 63d. : £128 7 6 :: 1m.;
or 63d. : 30810d. :: 1m. : 489m. 47 $\frac{1}{2}$ r.

Exer.

12. 65*d.* : £583 3 4 :: 1*m.*;
or 65*d.* : 139960*d.* :: 1*m.* : 2153*m.* 230½*r.*+5.
13. 1 *m.* : 482 *m.* 615 *r.* :: 62½*d.*;
or 1000*r.* : 482615*r.* :: 62½*d.* : £125 13 7½+.
14. 1 *m.* : 765*m.* 15*r.* :: 64½*d.*;
or 1000*r.* : 765015*r.* :: 64½*d.* : £204 16 0+.
15. 11*fl.* 95*c.* : 386*fl.* 108*c.* :: £1;
or 4495*c.* : 154508*c.* :: £1 : £34 7 5½+.
16. 12*fl.* 200*c.* : 9815*fl.* :: £1;
or 5000*c.* : 3926000*c.* :: £1 : £785 4 0.
17. £1 : £100 :: 11*fl.* 56*c.*;
or £1 : £100 :: 4456*c.* : 1114*fl.*
18. £1 : £513 6 8 :: 12*fl.* 25*c.*;
or 240*d.* : 123200*d.* :: 4825*c.* : 6192*fl.* 33½*c.*
19. 16*m.* 9*sch.* : 827*r.* 2*m.* 12*sch.* 8*p.* :: £1;
or 3180*p.* : 476888*p.* :: £1 : £149 19 3½+600.
20. £1 : £586 11 6 :: 16*m.* 15*s.* 6*p.*;
or 240*d.* : 140778*d.* :: 3258*p.* : 3317*r.* 2*m.* 7*s.* 1½*p.*
21. 4*r.* 18*gr.* : 1726*r.* 15*gr.* 6*p.* :: £1;
or 1368*p.* : 497274*p.* :: £1 : £363 10 1+72.
22. 1*r.* : 518*r.* 5*m.* 10*s.* :: 4*s.* 10*d.*;
or 96*s.* : 49818*s.* :: 58*d.* : £125 8 2½.
23. £1 : £500 :: 25*l.* 80*c.*;
or £1 : £500 :: 2580*c.* : 12900 *lire.*
24. 1*sc.* : 2760*sc.* 9*j.* 8*bo.* :: 6*s.* 10*d.*;
or 120*bo.* : 331298*bo.* :: 82*d.* : £943 5 6¾+104.
25. 2*s.* 4*d.* : £1000 :: 1*r.*;
or 28*d.* : 240000*d.* :: 1*r.* : 8571*r.* 6*a.* 10*p.*+8.
26. 4*s.* 8*d.* : £275 :: 1*dol.*;
or 56*d.* : 66000*d.* :: 1*dol.* : 1178*dol.* 5*di.* 7*c.*+8.
27. £116 : £100 :: £1645 15 0 : £1418 15 0.

ALLIGATION.

(Arithmetic, page 137.)

- | | | | | |
|----|-----------|------|--------|--|
| 1. | 60 gals. | × 24 | = | 1440 |
| | 120 „ | × 28 | = | 3360 |
| | | | | |
| | 180 gals. | : | 1 gal. | :: 4800 <i>s.</i> : 26 <i>s.</i> 8 <i>d.</i> |

Exer.

$$9. \quad 5\frac{1}{2}s. \left\{ \begin{array}{l} 3\frac{1}{2} \\ 4 \\ 6 \\ 7\frac{1}{2} \end{array} \right. \begin{array}{l} \frac{1}{2} \\ 2 \\ 2 \\ 1\frac{1}{2} \end{array}$$

$$6 : 240 :: \frac{1}{2} : 20 \text{ at } 3s. 6d.$$

$$:: 2 : 80 \text{ at } 4s. 0d.$$

$$:: 2 : 80 \text{ at } 6s. 0d.$$

$$:: 1\frac{1}{2} : 60 \text{ at } 7s. 6d.$$

$$10. \quad 45s. \left\{ \begin{array}{l} 35s. \\ 47\frac{1}{2}s. \\ 52\frac{1}{2}s. \end{array} \right. \begin{array}{l} 2\frac{1}{2} + 7\frac{1}{2} = 10 \\ 10 \\ 10 \end{array}$$

$$\overline{30} : 80 :: 10 : 26a. 2r. 26\frac{2}{3} \text{ per each.}$$

REDUCTION OF FRACTIONS.

(Arithmetic, page 151.)

Exer.

1. Divide both terms by 6

2. " " " 6

3. " " " 72

4. " " " 2

5. " " " 5

6. " " " 3

7. " " " 256

Exer.

8. Divide both terms by 4

9. " " " 13

11. " " " 9

12. " " " 8

14. " " " 2

15. " " " 9

16. In finding the common denominator, multiply the three denominators continually together.
17. Reject 3 and 4, and multiply 8 and 9.
18. Reject all the denominators except the third.
19. Reject 20 and divide by 10; the product of all the quotients by 10, is the denominator.
20. The product of the denominators is the common denominator.
21. Reject 11 and divide by 11; the product of the quotients by 11 is the common denominator.
22. Reject 5, divide by 4 and 21; the product of quotients by 4 and 21 is the denominator.
23. Reject 2 and 5 and divide by 9; the product of the quotients by 9 is the denominator.

Exer.

24. Divide by 3; the product of the quotients by 3 is the denominator.

25. Divide the numerator by the denominator.

30. $8 \times 3 + 2 = 26$ the numerator, and 3 the denominator.

31. $215 \times 29 + 28 = 6263$ the numerator, and 29 the denominator.

32. $19 \times 11 + 8 = 217$, the numerator, and 11 the denominator.

34. 1s. $8\frac{1}{2}d.$ = 41 halfpence, and 2s. 6d. = 60 halfpence; then $\frac{41}{60}$.

35. 18s. $5\frac{1}{2}d. + \frac{2}{3} = 2217$ fifths of halfpence, and £1 = 2400 fifths of halfpence; then $\frac{2217}{2400} = \frac{739}{800}$.

36. 3 ft. 11 in. = 47 in., and 1 p. = 198 in.; then $\frac{47}{198}$.

37. 13 hrs. 3 min. 15 sec. = 46995 sec.; and 1 day = 86400 sec.; then $\frac{46995}{86400} = \frac{3133}{5760}$.

38. 2 qrs. 17 lbs. = 73 lbs., and 1 cwt. = 112; then $\frac{73}{112}$.

39. $7^\circ 24'' = 25224''$, and $90^\circ = 324000''$; then $\frac{25224}{324000} = \frac{1051}{13500}$.

40. 2 a. 3 r. 15 p. = 455 p., and 84 a. 3 r. 18 p. = 13578 p. then $\frac{455}{13578}$.

41. $3s. \div 4 = 9d.$

42. $\pounds 11 \div 18 = 12s. 2\frac{2}{3}d.$

43. $4 m. \div 5 = 4224$ feet.

44. $7 \text{ leagues} \div 8 = 4620$ yards.

45. $\pounds 27 \text{ } 11 \text{ } 6 \times 11 \div 12 = \pounds 25 \text{ } 5 \text{ } 6\frac{1}{2}$.

46. $5 \text{ cwt.} \div 7 = 2 \text{ qrs. } 24 \text{ lbs.}$

47. $9 \text{ lbs.} \div 19 = 5\frac{1}{19} \text{ oz.}$

48. $7 a. \div 8 = 3 r. 20 p.$

49. $3 \text{ crowns} \div 4 = 3s. 9d.$

50. $3 \text{ days} \div 10 = 7 \text{ hrs. } 12 \text{ min.}$

51. $5 \text{ days} = 120 \text{ hrs.}$, and $\frac{120}{6} = 20$.

52. $15s. = 720$ farthings, and $\frac{720}{24} = 30$.

53. $4 \text{ guineas} = 4032$ farthings, and 1s. = 48 farthings; $48 \times 5 = 240$; then $\frac{4032}{240} = 16\frac{4}{5}$.

54. $6 \text{ cwt.} = 48 \text{ st.}$; then $\frac{48}{7}$.

55. $1 m. = 5280 \text{ ft.}$; then $\frac{5280}{12} = 440$.

56. $8 a. = 1280 p.$; then $\frac{1280}{5}$.

ADDITION OF FRACTIONS.

(Arithmetic, page 156.)

Exer.

1. Com. denom. 72 ; numer. 36, 54, 45, 32, 42, 44, 39.
 2. „ „ 2800 ; „ 1120, 1225, 392, 60, 3.
 3. „ „ 24 ; „ 24, 18, 20, 21.
 4. „ „ 3465 ; „ 693, 1925, 990, 2520.
 5. „ „ 12 ; „ 5, 2, 6, 8,
 6. „ „ 18 ; „ 12, 8, 15.
 7. „ „ 18 ; „ 9, 12, 10.
 8. „ „ 210 ; „ 84, 140, 105, 120.
 9. „ „ 105 ; „ 15, 42, 28.
 10. „ „ 128 ; „ 64, 32, 16, 8, 4, 2, 1.
-

MULTIPLICATION OF FRACTIONS.

(Arithmetic, page 157.)

6. The improper fractions are $\frac{11}{2}$ and $\frac{11}{4}$.
 7. „ „ $\frac{191}{10}$ „ $\frac{77}{5}$.
 8. „ „ $\frac{11}{5}$, $\frac{5}{1}$, „ $\frac{7}{3}$.
 9. „ „ $\frac{58}{7}$ „ $\frac{153}{14}$.
 10. „ „ $\frac{5}{1}$ „ $\frac{158}{30}$.
14. Here add the two first, and multiply the sum by the product of the two last.
 15. Multiply the sum of the two last by 5.
 16. Multiply the sum of the three first by the product of the two last.
 17. Multiply the product of the three first by the last.
 18. Multiply the sum of the two first by the product of the two last.

SUBTRACTION OF FRACTIONS.

(Arithmetic, page 158.)

Exer.

1. Common denominator 18; numerator 17, 15.
2. " " 6; " 4, 3.
3. " " 9; " 4, 6.
4. " " 42; " 15, 34.
5. Here find the sum of the two first, and the product of the two last, then the common denominator, 21; numerator 7, 1.
6. $\frac{2}{3} \times 5 = \frac{10}{3}$ and $(4\frac{1}{2} - 2\frac{2}{3} = \frac{11}{6}) \times \frac{5}{8} = \frac{55}{48}$; then $\frac{10}{3} = \frac{100}{30}$ and $\frac{55}{48} - \frac{11}{6} = \frac{105}{48} = 2\frac{1}{16}$.
7. $7\frac{1}{2} + 4\frac{1}{2} = 12\frac{1}{2}$, and $2\frac{1}{2} \times 3\frac{1}{2} = 8\frac{1}{4}$; and $12\frac{1}{2} - 8\frac{1}{4} = 4\frac{1}{4}$.
8. $\frac{5}{8} \times \frac{7}{8} = \frac{35}{64}$, and $\frac{4}{5} \times \frac{7}{8} = \frac{28}{40}$; then $\frac{35}{64} - \frac{28}{40} = \frac{17}{640}$.
9. $11\frac{1}{2} - 10\frac{1}{2} \times 5\frac{1}{2} = \frac{19}{2}$, and $\frac{7}{8} \times 3\frac{1}{2} = \frac{19}{8}$; then $\frac{19}{2} - \frac{19}{8} = \frac{19}{4} = 4\frac{3}{4}$.
10. $\frac{3}{4} \times \frac{1}{2} \times \frac{1}{11} = \frac{1}{88}$, and $\frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$; then $\frac{7}{16} - \frac{1}{88} = \frac{309}{880}$.

DIVISION OF FRACTIONS.

(Arithmetic, page 159.)

3. The improper fractions are $\frac{2}{3}$ and $\frac{4}{5}$.
4. " " " $\frac{4}{8}$ " $\frac{10}{10}$.
5. " " " $\frac{1}{2}$ " $\frac{4}{4}$.
6. " " " $\frac{7}{8}$ " $\frac{11}{8}$.
7. " " " $\frac{7}{2}$ " $\frac{3}{2}$.
8. " " " $1\frac{3}{8}$ " $1\frac{3}{8}$.
9. " " " $\frac{7}{2}$ " $\frac{1}{2}$.
10. " " " $1\frac{1}{2}$ " $1\frac{1}{2}$.
11. " " " $\frac{7}{8}$ " $\frac{1}{8}$.
12. " " " $\frac{3}{4}$ " $\frac{2}{4}$.
13. " " " $\frac{1}{2}$ " $\frac{7}{8}$.

Exer.

14. By reducing the compound fractions this becomes $\frac{11}{11} \div \frac{11}{11}$.
15. By reducing these fractions they become $\frac{11}{11} \div \frac{11}{11}$.
16. By reduction these become $\frac{1}{1} \div \frac{1}{1}$.
17. By reduction these become $\frac{11}{11} \div \frac{11}{11}$.
18. By reduction these become $\frac{11}{11} \div \frac{11}{11} = \frac{1}{1} \div \frac{1}{1}$.
19. By reduction these become $\frac{1}{1} \div \frac{1}{1}$.
20. The quotient of the two first $= \frac{11}{11} \div \frac{11}{11}$.
21. $\frac{1}{1}$ of $2\frac{1}{1} \div 7\frac{1}{1} = \frac{1}{1}$, and
 $1\frac{1}{1} \div 3 = \frac{1}{1}$; then
 $\frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$; again
 $\frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$; and $\frac{1}{1} \div 5\frac{1}{1} \div 2 =$
 $\frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$; then
 $\frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$; and $\frac{1}{1} \div \frac{1}{1} = 2\frac{1}{1}$.
22. $2\frac{1}{1} \div 6\frac{1}{1} \div 7 = \frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$, and $\frac{1}{1} \div 5\frac{1}{1} \div 3 =$
 $\frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$; again
 $\frac{1}{1}$ of $6\frac{1}{1} = \frac{1}{1}$, and $\frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$, and
 $\frac{1}{1} \div \frac{1}{1}$ of $\frac{1}{1} = \frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$; then
 $\frac{1}{1} \div \frac{1}{1} = \frac{1}{1}$, and
 $\frac{1}{1} \div \frac{1}{1} = \frac{1}{1} \div \frac{1}{1} = 3\frac{1}{1}$.

PROPORTION IN FRACTIONS.

(Arithmetic, page 160.)

1. £ $\frac{1}{1}$: $\frac{1}{1}$ s. :: 76 yds.;
 or $\frac{1}{1}$ s. : $\frac{1}{1}$ s. :: $\frac{1}{1}$ yds. : 3 $\frac{1}{1}$ yds.
2. 1 oz. : 16 $\frac{1}{1}$ oz. :: 6s. 4d. = 6 $\frac{1}{1}$ s.;
 or $\frac{1}{1}$ oz. : 1 $\frac{1}{1}$ oz. :: $\frac{1}{1}$ s. : £5 6 10 $\frac{1}{1}$.
3. 5 $\frac{1}{1}$ hs. : 6 $\frac{1}{1}$ hs. :: 17 $\frac{1}{1}$ days;
 or $\frac{1}{1}$ hs. : $\frac{1}{1}$ hs. :: $\frac{1}{1}$ ds. : 20 $\frac{1}{1}$ days.
4. 10 $\frac{1}{1}$ lbs. : 15 $\frac{1}{1}$ lbs. :: 8 $\frac{1}{1}$ s.;
 or $\frac{1}{1}$ lbs. : $\frac{1}{1}$ lbs. :: $\frac{1}{1}$ s. : 12s. 4 $\frac{1}{1}$ d.
5. $\frac{1}{1}$ fls. : $\frac{1}{1}$ fls. } :: 12 men : 31 $\frac{1}{1}$ men.
 $2\frac{1}{1}$ ds. : 3 $\frac{1}{1}$ ds. }
6. 5 $\frac{1}{1}$ d. : 7 $\frac{1}{1}$ d. } :: 2 $\frac{1}{1}$ lbs. : 3 lbs. 1 $\frac{1}{1}$ oz.
 £1 $\frac{1}{1}$: £1 $\frac{1}{1}$ }

DECIMAL FRACTIONS.

ADDITION.

(Arithmetic, page 169.)

Exer.

$$\begin{array}{r}
 1. \quad 5\cdot27 \\
 726\cdot015 \\
 \quad \cdot0008 \\
 \quad \cdot56 \\
 \hline
 731\cdot84508
 \end{array}$$

$$\begin{array}{r}
 3. \quad 15\cdot2 \\
 \quad \cdot004 \\
 \quad \cdot6666 \\
 283\cdot2143 \\
 50000\cdot \\
 \hline
 50299\cdot0849
 \end{array}$$

$$\begin{array}{r}
 3. \quad \cdot00032 \\
 \cdot00027 \\
 \cdot009 \\
 \cdot00003 \\
 \cdot00038 \\
 \hline
 \cdot01
 \end{array}$$

Exer.

$$\begin{array}{r}
 4. \quad \cdot0300003 \\
 3\cdot000003 \\
 \cdot003 \\
 30\cdot \\
 \cdot00033 \\
 \cdot3 \\
 \hline
 33\cdot3'
 \end{array}$$

$$\begin{array}{r}
 5. \quad 417\cdot \\
 \quad \cdot2543'33 \\
 \quad \cdot156'477 \\
 \quad \cdot013433 \\
 582\cdot57'5757 \\
 \hline
 1000\cdot
 \end{array}$$

MULTIPLICATION OF DECIMALS.

INVOLUTION II.

(Arithmetic, page 170.)

Exer.

$$\begin{array}{r}
 1. \quad \cdot958 \\
 3\cdot4 \\
 \hline
 3832 \\
 2874 \\
 \hline
 3\cdot2572
 \end{array}$$

Exer.

$$\begin{array}{r}
 2. \quad 5\cdot0083 \\
 6\cdot4 \\
 \hline
 200332 \\
 300498 \\
 \hline
 32\cdot05312
 \end{array}$$

Exer.

$$\begin{array}{r} 3. \quad .7135 \\ \quad .00018 \\ \hline \quad 57080 \\ \quad 7135 \\ \hline .000128430 \end{array}$$

$$\begin{array}{r} 4. \quad 4.356 \\ \quad 700.8 \\ \hline \quad 34848 \\ \quad 30492 \\ \hline 3052.6848 \end{array}$$

$$\begin{array}{r} 5. \quad 3.007 \\ \quad .36 \\ \hline \quad 18042 \\ \quad 9021 \\ \hline 1.08252 \end{array}$$

$$\begin{array}{r} 6. \quad .030256 \\ \quad 84. \\ \hline \quad 121024 \\ \quad 242048 \\ \hline 2.541504 \end{array}$$

Exer.

$$\begin{array}{r} 7. \quad 500.83 \\ \quad 121. \\ \hline \quad 50083 \\ \quad 100166 \\ \hline \quad 50083 \\ 60600.43 \end{array}$$

$$\begin{array}{r} 8. \quad .0002 \\ \quad 100. \\ \hline .0200 \end{array}$$

$$\begin{array}{r} 9. \quad 85.9 \\ \quad 6.6 \\ \hline \quad 5154 \\ \quad 5154 \\ \hline 566.94 \end{array}$$

$$\begin{array}{r} 10. \quad .04081 \\ \quad .00057 \\ \hline \quad 28567 \\ \quad 20405 \\ \hline .0000232617 \end{array}$$

BY CONTRACTION.

Exer.

$$\begin{array}{r} 11. \quad 38.7453 \\ \quad 2646.0 \\ \hline \quad 23247+ \\ \quad 1937+ \\ \quad 233- \\ \quad 8- \\ \hline 25.425+ \end{array}$$

Exer.

$$\begin{array}{r} 13. \quad .07 = .0000016807 \\ \quad 3.0 \\ \hline .0000005 \end{array}$$

DIVISION OF DECIMALS

Exer.

12. $5 \cdot 17^2 = 26 \cdot 7289000$

$4 \cdot 07^2 = 1116 \cdot 79136 ;$

then :—

$26 \cdot 7289000$

$63197 \cdot 6111$

$\underline{267289000}$

26728900

2672890

1603734

187102

24056

267

80

16

$\underline{29850 \cdot 6045}$

14. $1 \cdot 4^2 = 14 \cdot 75789056$

$\cdot 013^2 = 000000028561$

then :—

$14 \cdot 7578905600$

$165820000000 \cdot 0$

$\underline{2952-}$

$1181-$

$74+$

$8+$

$\underline{0000004215}$

Exer.

15. $1 \cdot 5^2 = 2 \cdot 25$

$\frac{3}{4}^2 = \frac{9}{16} = .4^2 ;$

then $2 \cdot 250000000000$

$444'44444444'0$

$\underline{-900}$

90

9

$\underline{\cdot 999' = 1 \cdot}$

16. $\frac{3}{4}^2 = .5625, \times 7 \cdot 3 =$

$4 \cdot 10625.$

$1 \cdot 25^2 = 3 \cdot 0517578125$

$52601 \cdot 4$

$\underline{1220703}$

30518

1831

61

15

$\underline{12 \cdot 53128}$

DIVISION OF DECIMALS.

— (Arithmetic, page 173.)

Exer.

1. $\cdot 7294)892 \cdot 5600($

2. $50 \cdot 0400)191 \cdot 5217($

3. $\cdot 0095)276 \cdot 0000($

4. $29 \cdot 55)318 \cdot 59($

5. $764 \cdot 333) \cdot 5'68'(0 \cdot$

6. $\cdot 4'04') \cdot 8'88($

Exer.

7. $\cdot 5')5 \cdot 0($

8. $\cdot 014745') \cdot 786400($

9. $\cdot 00009)5 \cdot 17200($

10. $5432 \cdot 1)987 \cdot 6(0 \cdot$

11. $\cdot 0003') \cdot 00009'(0 \cdot$

Exer.

$$\begin{array}{r}
 4. \quad 2955 \overline{) 31859(10 \cdot 78 -} \\
 \underline{2955} \\
 2309 \\
 \underline{2068} \\
 241 \\
 \underline{236} \\
 5
 \end{array}$$

Exer.

$$\begin{array}{r}
 6. \quad 4'04' \cdot 888(2' \cdot 200' \\
 \underline{808} \\
 808 \\
 \underline{808} \\
 8, \text{ \&c.}
 \end{array}$$

12.

$$\left(\left(\frac{4}{3}\right)^3 = \frac{64}{27}\right) - \left(\left(\frac{2}{3}\right)^3 = \frac{8}{27}\right) = \frac{56}{27} =$$

$$\left(\frac{7}{9} \text{ of } \frac{3}{8}\right)^3 = \left(\frac{7}{24}\right)^3 = \frac{49}{576} =$$

$$\text{then } 44 \div \frac{49}{576} = \frac{44 \cdot 576}{49} = 24 \cdot 3'80952'.$$

PRACTICE.

(Arithmetic, page 176.)

				£	s.	d.
11.	Price at £5,	215	0	0
	3s. 4d. = $\frac{1}{3}$ of £5,	7	3	4
12.	£3,	759	0	0
	15s. = $\frac{1}{4}$ of £3,	189	15	0
	2s. 6d. = $\frac{1}{8}$ of 15s.	31	12	6
13.	10s.	243	0	0
	2·6,	60	15	0
	·4,	6	1	6
	·1,	2	0	6
14.	1s.	70	4	0
	1½d.	8	15	6
15.	6d.	19	17	6
	3d.	9	18	9
	1d.	3	6	3
16.	£1,	269	0	0
	10s.	134	10	0
	5s.	67	5	0
	1s.	13	9	0

Ex.	Price at.	£	s.	d.	Ex.	Price at.	£	s.	d.
17.	5s. ...	28	0	0	26.	£1, ...	7649	0	0
	1s. ...	5	12	0		4d. = $\frac{1}{10}$ of £1, ...	127	9	8
	6d. ...	2	16	0		1d. ...	31	17	5
	3d. ...	1	8	0		$\frac{1}{2}$ d. ...	15	18	$8\frac{1}{2}$
18.	£3, ...	2451	0	0		$\frac{1}{4}$ d. ...	7	19	$4\frac{1}{4}$
	6s. 8d. ...	272	6	8	27.	£1, ...	7841	0	0
	8d. ...	27	4	8		10s. ...	3920	10	0
19.	6d. ...	3	14	6		5s. ...	1960	5	0
	$1\frac{1}{2}$ d. ...	0	18	$7\frac{1}{2}$		$\frac{1}{2}$ d. ...	16	6	$8\frac{1}{2}$
	$\frac{1}{4}$ d. ...	0	3	$1\frac{1}{4}$	28.	£7, ...	2961	0	0
20.	10s. ...	112	0	0		10s. ...	211	10	0
	1s. 3d. ...	14	0	0		1s. 8d. ...	35	5	0
	$1\frac{1}{2}$ d. ...	1	8	0		$2\frac{1}{2}$ d. ...	4	8	$1\frac{1}{2}$
21.	6s. 8d. ...	324	0	0		$1\frac{1}{4}$ d. ...	2	4	$0\frac{1}{4}$
	1s. 8d. ...	81	0	0	29.	10s. ...	28	10	0
	5d. ...	20	5	0		5s. ...	14	5	0
22.	£2, ...	730	0	0		2s. 6d. ...	7	2	6
	5s. ...	91	5	0		1s. 3d. ...	3	11	3
	1s. 8d. ...	30	8	4		$7\frac{1}{2}$ d. ...	1	15	$7\frac{1}{2}$
	5d. ...	7	12	1	30.	10s. ...	730	0	0
	$\frac{1}{2}$ d. ...	0	15	$2\frac{1}{2}$		5s. ...	365	0	0
23.	£1, ...	114	0	0		2s. 6d. ...	182	10	0
	10s. ...	57	0	0		3d. ...	18	5	0
	6s. 8d. ...	38	0	0		$1\frac{1}{2}$ d. ...	9	2	6
	1s. 8d. ...	9	10	0	31.	£1, ...	1728	15	0
	$2\frac{1}{2}$ d. ...	1	3	9		£3, ...	5186	5	0
24.	£3, ...	279	0	0		5s. ...	432	3	9
	5s. ...	23	5	0		4d. ...	28	16	3
	2s. 6d. ...	11	12	6	32.	£1, ...	312	15	$6\frac{2}{3}$
	3d. ...	1	3	3		£2, ...	625	11	$1\frac{1}{3}$
	$1\frac{1}{2}$ d. ...	0	11	$7\frac{1}{2}$		10s. ...	156	7	$9\frac{1}{3}$
25.	6d. ...	127	13	6		5s. ...	78	3	$10\frac{2}{3}$
	3d. ...	63	16	9		2s. 6d. ...	39	1	$11\frac{1}{3}$
	$\frac{3}{4}$ d. ...	15	19	$2\frac{1}{4}$		3d. ...	3	18	$2\frac{2}{3}$

Ex.	Price at.	£	s.	d.
33.	£1, ...	59	3	9
	5s. ...	14	15	11½
	2s. 6d....	7	7	11½
	1s. 3d....	3	13	11½

34.	£1, ...	153	18	0
	5s. ...	38	9	6
	4s. ...	30	15	7½
	5d. = ⅙ of 5s.	3	4	1½

35.	£1, ...	98	18	4
	3s. 4d....	16	9	8½
	5d. ...	2	1	2½
	½d. ...	0	4	1½

36.	£1, ...	387	16	0
	10s. ...	193	18	0
	2s. 6d....	48	9	6
	3d. ...	4	16	11½
	1d. ...	1	12	3½

37.	£1, ...	76	14	0
	10s. ...	38	7	0
	5s. ...	19	3	6
	2s. 6d....	9	11	9
	1s. 3d....	4	15	10½

38.	£1, ...	159	7	1½
	10s. ...	79	13	6½
	1s. 3d....	9	19	2½
	5d. ...	3	6	4¾
	½d. ...	0	6	7¾

39.	£1, ...	236	19	0
	2s. ...	23	13	10½
	1s. ...	11	16	11½
	4d. ...	3	18	11½
	1d. ...	0	19	8½

Ex.	Price at.	£	s.	d.
40.	£1, ...	123	12	6
	½d. = ⅙ of £1,	0	2	6½
		123	9	11½

41.	£1, ...	57	17	6
	5s. ...	14	9	4½
	2s. ...	5	15	9
	4d. ...	0	19	3½
	½d. ...	0	2	4½

42.	£1, ...	11	14	1½
	£5, ...	58	10	6½
	2s. 6d....	1	9	3½
	1s. 3d....	0	14	7½
	2½d. ...	0	2	5½

43.	£1, ...	37	15	0
		188	15	0
	½d. = ⅙ of £1,	0	0	9½
		188	14	2½

44.	£1, ...	175	18	11½
	10s. ...	87	19	5½
	5s. ...	43	19	8½
	2s. ...	17	11	10½
	4d. ...	2	18	7½

45.	£1, ...	42	2	10½
	£5, ...	210	14	3½
	10s. ...	21	1	5½
	6s. 8d....	14	0	11½

46.	£1, ...	34	12	6
	£2, ...	69	5	0
	10s. ...	17	6	3
	1s. ...	1	14	7½
	6d. ...	0	17	3½

Ex.	Price at.	£	s.	d.	Ex.	Price at.	£	s.	d.	
47.	£1, ...	31	4	4	51.	£1, ...	56	18	9	
	6d. = $\frac{1}{20}$ of £1, 0	15	7	$\frac{7}{10}$	5s. ...	14	4	8	$\frac{1}{4}$	
		30	8	8 $\frac{1}{8}$	4d. ...	0	18	11	$\frac{3}{4}$	
48.	£1, ...	467	19	9	52.	£1, ...	14	2	6	
	£3, ...	1403	19	3		£2, ...	28	5	0	
	10s. ...	233	19	10 $\frac{1}{2}$	6d. ...	0	7	0	$\frac{3}{4}$	
	5s. ...	116	19	11 $\frac{1}{4}$	53.	£1, ...	98	10	0	
	2s. 6d. ...	58	9	11 $\frac{5}{8}$		10s. ...	49	5	0	
	6d. ...	11	13	11 $\frac{3}{8}$	6s. 8d. ...	32	16	8		
49.	£1, ...	96	13	0	2s. 6d. ...	12	6	3		
	£3, ...	289	19	0	6d. ...	2	9	3		
	10s. ...	48	6	6	$\frac{1}{2}$ d. ...	0	4	1	$\frac{1}{4}$	
	6s. 8d. ...	32	4	4	54.	£1, ...	76	7	6	
	1s. 8d. ...	8	1	1		3d. ...	0	19	1	$\frac{1}{8}$
	2d. ...	0	16	1 $\frac{3}{10}$	$\frac{1}{4}$ d. ...	0	1	7	$\frac{3}{4}$	
50.	£1, ...	44	13	1 $\frac{1}{2}$	55.	£1, ...	144	1	3	
	£55, ...	2456	1	10 $\frac{1}{2}$		10s. ...	72	0	7	$\frac{1}{2}$
	10s. ...	22	6	6 $\frac{3}{4}$	6s. 8d. ...	48	0	5		
	5s. ...	11	3	3 $\frac{3}{8}$	1s. 8d. ...	12	0	1	$\frac{1}{4}$	
	1s. 3d. ...	2	15	9 $\frac{3}{4}$	5d. ...	3	0	0	$\frac{1}{8}$	
	3d. ...	0	11	1 $\frac{3}{4}$						
	1 $\frac{1}{2}$ d. ...	0	5	6 $\frac{3}{4}$						

56.	lbs.	oz.	dwt.	grs.	
	12,	9,	15,	12,	at £3 17 6.
	3				
	£36	0	0		= price of 12 lb. at £3.
	10s. = £ $\frac{1}{2}$ - 6	0	0		= „ „ at 10s.
	5s. = $\frac{1}{2}$ - 3	0	0		= „ „ at 5s.
	2s. 6d. = $\frac{1}{2}$ - 1	10	0		= „ „ at 2s. 6d.
	6 oz. = $\frac{1}{2}$ lb. - 1	18	9		= „ 6 oz. at £3 17 6.
	3 oz. = $\frac{1}{2}$ - 0	19	4 $\frac{1}{2}$		= „ 3 oz.
	15 dwt. = $\frac{1}{4}$ - 0	4	10 $\frac{1}{8}$		= „ 15 dwt.
	12 grs. = $\frac{1}{20}$ - 0	0	1 $\frac{1}{4}$		= „ 12 grs.
		£49	13	1 $\frac{1}{8}$	

Exer.

57.

oz.	dwt.	grs.
10,	9,	18,

 at £0 12 3 per oz.

$10s. = £\frac{1}{2} - £5$ 0 0 = price of 10 oz. at 10s. per oz.
 $2s. = \frac{1}{8} -$ 1 0 0 = „ „ at 2s. „
 $3d. = \frac{1}{8} -$ 0 2 6 = „ „ at 3d. „

$5 \text{ dwt.} = \frac{1}{4} \text{ oz.} -$ 0 3 $0\frac{3}{4}$ = 5 dwt. at 12s. 3d.
 $4 \text{ dwt.} = \frac{1}{5} \text{ oz.} -$ 0 2 $5\frac{2}{5}$ = 4 „ „
 $12 \text{ grs.} = \frac{1}{8} -$ 0 0 $3\frac{3}{8}$ = 12 grs. „
 $6 \text{ grs.} = \frac{1}{2} -$ 0 0 $1\frac{7}{8}$ = 6 „ „

£6 8 $5\frac{3}{8}$

58.

lbs.	oz.	dwt.	grs.
56,	6,	0,	6,

 at £3 5 4 per lb.

3

$£168$ 0 0 = price of 56 lb. at £3.
 $5s. = £\frac{1}{4} -$ 14 0 0 = „ „ at 5s.
 $4d. = \frac{1}{12} -$ 0 18 8 = „ „ at 4d.

$6 \text{ oz.} = \frac{1}{4} \text{ lb.} -$ 1 12 8 = 6 oz. at £3 5 4 per lb.
 $6 \text{ grs.} = \frac{1}{20} -$ 0 0 $0\frac{3}{5}$ = 6 grs.

£184 11 $4\frac{3}{5}$

59.

£5	8	9
		8
<hr/>		
43	10	0
		9
<hr/>		
£391	10	0

 = price of 72 lb. at £5 8 9.

$1 \text{ gr.} = \frac{1}{5760} \text{ lb.} -$ 0 0 $0\frac{29}{128}$ = „ 1 gr.

£391 9 $11\frac{99}{128}$

60.

oz.	dwt.
8,	15,

 at £1 3 4 per oz.

8

9 6 8 = price of 8 oz.
 $10 \text{ dwt.} = \frac{1}{2} \text{ oz.} -$ 0 11 8 = „ 10 dwt.
 $5 \text{ dwt.} = \frac{1}{4} -$ 0 5 10 = „ 5 dwt.

£10 4 2

Ex.	Price at.	£	s.	d.	Ex.	Price at.	£	s.	d.
61.	£1, ...	87	17	6	68.	£1, ...	58	10	0
	10s. ...	43	18	9		10s. ...	29	5	0
	5s. ...	21	19	4½		1s. ...	2	18	6
	2s. 6d....	10	19	8¼		¼d. ...	0	1	2½
62.	£1, ...	173	5	9	69.	£1, ...	143	15	0
	2s. 6d....	21	13	2½		10s. ...	71	17	6
	3d. ...	2	3	3⅞		6s. 8d....	47	18	4
						1s. 8d....	11	19	7
63.	£1, ...	48	12	6		2½d. ...	1	9	11⅞
	6s. 8d....	16	4	2		1¼d. ...	0	14	11⅞
	2s. 6d....	6	1	6¾	70.	£1, ...	95	17	6
	8d. ...	1	12	5		10s. ...	47	18	9
64.	£1, ...	97	15	8¼		5s. ...	23	19	4½
	10s. ...	48	17	10⅞		4d. ...	1	11	11½
	4s. ...	19	11	1⅞		½d. ...	0	3	11⅞
	4d. = ⅓ of 4s.	1	12	7¼	71.	£1, ...	23	19	9⅞
	1d. ...	0	8	1⅞		10s. ...	11	19	10⅞
65.	£1, ...	56	6	0		6s. 8d....	7	19	11½
	10s. ...	28	2	6		8d. ...	0	15	11⅞
	1s. 3d....	3	10	3¾	72.	£1, ...	15	19	4½
	5d. ...	1	3	5¼		£2, ...	31	18	9
	¼d. ...	0	2	4½		10s. ...	7	19	8¼
66.	£1, ...	17	15	0		6s. 8d....	5	6	5½
	10s. ...	8	17	6		1s. 8d....	1	6	7⅞
	3s. 4d....	2	19	2		5d. ...	0	6	7⅞
	1d. ...	0	1	5¾		1d. ...	0	1	3⅞
67.	£1, ...	827	15	0		½d. ...	0	0	7⅞
	10s. ...	413	17	6	74.	£1, ...	57	19	11⅞
	3s. 4d....	137	19	2		10s. ...	28	19	11⅞
	10d. ...	34	9	9½		6s. 8d...	19	6	7⅞
	5d. ...	17	4	10¾		1s. 8d...	4	16	7⅞
	¼d. ...	1	14	5⅞		5d. ...	1	4	1⅞
						2d. ...	0	9	7⅞

Exer.

73. 31 lb. 11 oz. 19 dwt. 23 grs. at £5 14 7½ per oz.

					12
Price of 1 lb.	...	68	15	6	
					4
„ 4 lb.	...	275	2	0	
					8
„ 32 lb.	...	2200	16	0	
off 1 gr. = $\frac{1}{7168}$ of 1 lb.	...	0	0	2½½	
		£2200	15	9½½	

75. 143 yds. 3 qrs. 3½ n., at £0 19 5½ per yard.

					12
Price of 12 yds.	...	11	13	3	
					12
„ 144 yds.	...	139	19	0	
„ ½ n. = ($\frac{1}{16}$ of yd.)		0	0	7½½	
		£139	18	4½½	

76. Price of 1 bar. ... £1 0 11

					12
„ 12 bar.	...	12	11	0	
					6
„ 72 bar.	...	75	6	0	
„ ½ st. = $\frac{1}{16}$ of 1 bar.		0	0	6½½	
		£75	5	5½½	

77. Price of 1 ton, ... £17 19 11½

					6
„ 6 ton,	...	107	19	9	
					9
„ 54 ton,	...	971	17	9	
„ ½ lb. = $\frac{1}{4480}$ of 1 ton,		0	0	0½½½	
		£971	17	8½½½	

Exer.

78. Price of 1 bar,	...	£0	9	11 $\frac{1}{4}$
				10
„ 10 bar.	...	4	19	9 $\frac{1}{4}$
				10
„ 100 bar.	...	49	17	11
„ $\frac{1}{2}$ st. = $\frac{1}{16}$ of 1 bar.		0	0	4 $\frac{1}{4}$
		£49	17	6 $\frac{1}{4}$

79.	47 yds. 3 qrs. 2 in.
	5
	235 0 0
2 qrs. = $\frac{1}{2}$ yd. ...	2 10 0
1 qr. = $\frac{1}{4}$...	1 5 0
2 in. = $\frac{1}{8}$ of 2 qrs..	0 5 6 $\frac{3}{4}$
price at £5. ...	239 0 6 $\frac{3}{4}$
$1\frac{1}{2}$ d. = $\frac{1}{16}$ of £5,	0 5 11 $\frac{1}{4}$
	£238 14 6 $\frac{3}{4}$

80. Price at £1,	...	£199	19	9 $\frac{3}{4}$
„ £10,	...	1999	18	2 $\frac{1}{4}$
„ off $\frac{1}{2}$ d. = $\frac{1}{16}$ of £1,		0	8	3 $\frac{3}{4}$
		£1999	9	10 $\frac{1}{4}$

TARE AND TRET,

(Arithmetic, page 190.)

			cwt.	qrs.	lbs.
1.	Gross,	...	66	1	20
	Tare,	...	2	2	5
	Net weight,	...	63	3	15
2.	Gross,	...	127	3	14
	Tare 7 lb. = $\frac{1}{16}$ of 1 cwt.,		7	3	27 $\frac{1}{2}$
	Net,	...	119	3	14 $\frac{1}{2}$

TARE AND TRET.

63

Exer.

3.	Gross per bag,	st.	lb.	
	Tare	56	10	
	Net per bag,	10½		
			55	13½	
			16		
	Net total,	893	6	
4.	Gross,	cwt.	qrs.	lbs.
	Tare 1 qr. 19 lbs.×12,	156	3	8
			5	0	4
			151	3	4
	Tret 2½ lb.=¼ of 100 lb.	...	3	3	5
	Net,	147	3	27
5.	Gross per fir,	cwts.	qrs.	lbs.
	Tare per fir,	2	1	3¼
			0	1	26
			1	3	5¼
					8
			14	1	14
	Tret 4 lb.=⅛ cwt.	...	0	2	1½
	„ 1 lb.=¼	...	0	0	14⅜
	Tret,	0	2	15⅞
	Net,	13	2	26½
7.	Gross per bale,	cwt.	qrs.	lbs.
	Tare,	3	0	3
			0	0	8
			2	3	23
	Tret 4 lb.=⅛ cwt.	...	0	0	11⅜
			2	3	11⅝
	Cloff 2 lbs.=⅙ of 3 cwt.,	...	0	0	1⅔
	Net per bale,	2	3	9⅓
					10
	Net 10 bales,	28	1	8⅞
					9
	Net total,	254	3	23⅞

Exer.

			cwts.	qrs.	lbs.
6.	Gross per hhd.	15	0	12
					12
	Gross total,	181	1	4
	Tare 2 qrs. 3 lb. $\times 12$,	6	1	8
			174	3	24
	Damage, 2 qrs. 18 lbs. $\times 4$,		2	2	16
			172	1	8
	8 lb. $= \frac{1}{12}$ cwt.	12	1	6 $\frac{2}{3}$
	4 lb. $= \frac{1}{6}$	6	0	17 $\frac{2}{3}$
	Tret,	18	1	23 $\frac{2}{3}$
	Total net,	153	3	12 $\frac{1}{4}$
	Price at £1, ...	£153	17	2	$\frac{1}{4}$ $\frac{5}{8}$
	„ at £2, ...	307	14	4	$\frac{3}{4}$ $\frac{5}{8}$
	„ at 10s. ...	76	18	7	$\frac{1}{8}$ $\frac{5}{8}$
	„ at 2s. 6d. ...	19	4	7	$\frac{2}{3}$ $\frac{2}{5}$ $\frac{5}{8}$

			cwt.	qrs.	lbs.
8.			7	3	4
					14
	Gross total,	109	0	0
	Tare,	1	2	14
			107	1	14
	Tret 2 $\frac{1}{2}$ lbs. $= \frac{1}{40}$ of 100 lb.		2	2	20 $\frac{1}{2}$ $\frac{3}{5}$
			104	2	21 $\frac{7}{10}$
	Cloff 2 lbs. $= \frac{1}{100}$ of 3 cwt,		0	2	13 $\frac{1}{10}$ $\frac{3}{5}$ $\frac{7}{10}$
	Net total,	104	0	7 $\frac{8}{10}$ $\frac{5}{10}$

9. 100 lbs. $- 3\frac{1}{2}$ lbs. $= 96\frac{1}{2}$ lbs.;
 then $96\frac{1}{2}$ lbs. : 35 cwt. 0 qr. $9\frac{1}{4}$ lb. :: 100 lbs. :
 4056 lbs.

112 lbs. $- 8$ lbs. $= 104$ lbs.; then

	lbs.	lbs.	cwt.	cwt.	qr.	lb.
	104	: 4056	::	1	: 39	0 0
Tare 7 lb. $\times 12$,	0	3	0
Gross total,	39	3	0
Gross per cask,	3	1	7

Exer.

10. £278 14 11 ÷ 16 = £17 8 $5\frac{3}{16}$ price of 1 tub.
 £2 4 4 : £17 8 $5\frac{3}{16}$:: 1 cwt. : 7 cwt. 3 qrs. $12\frac{1}{8}$ lb.
 336 lbs. - 2 lb. = 334 lbs.; then
 334 lbs. : 336 lbs. :: 7 cwt. 3 qrs. $12\frac{1}{8}$ lb. :
 7 cwt. 3 qrs. $17\frac{1}{2}$ lbs.;
 and 7 cwt. 3 qrs. $17\frac{1}{2}$ lbs. + 7 lb. tare, =
 7 cwt. 3 qr. $24\frac{1}{2}$ lb. = the gross weight per tub.

EVOLUTION.

(Arithmetic, page 196.)

Exer.

1. $\begin{array}{r} 3178 \cdot 215 \\ 25 \overline{) 56 \cdot 375} \text{ Answ.} \\ 106 \overline{) 678} \\ 636 \\ \hline 1123 \overline{) 4221} \\ 3369 \\ \hline 11267 \overline{) 85250} \\ 78869 \\ \hline 112745 \overline{) 638100} \\ 563725 \\ \hline \text{rem. } \cdot 074375 \end{array}$

2. $\begin{array}{r} \cdot 79, 18, 20 \\ 64 \overline{) 8898} \text{ Answ.} \\ 168 \overline{) 1518} \\ 1344 \\ \hline 1769 \overline{) 17420} \\ 15921 \\ \hline 17788 \overline{) 149900} \\ 142304 \\ \hline \text{rem. } \cdot 00007596 \end{array}$

Exer.

3. $\begin{array}{r} 2, 17 \cdot 98, 45 \\ 1 \overline{) 14 \cdot 76429} \text{ Ans.} \\ 24 \overline{) 117} \\ 96 \\ \hline 287 \overline{) 2198} \\ 2009 \\ \hline 2946 \overline{) 18945} \\ 17676 \\ \hline 29524 \overline{) 126900} \\ 118096 \\ \hline 295282 \overline{) 880400} \\ 590564 \\ \hline 2952849 \overline{) 28983600} \\ 26575641 \\ \hline \text{rem. } \cdot 0002407959 \end{array}$
4. $\begin{array}{r} \cdot 3980 \\ 36 \overline{) 6308} \text{ Answ.} \\ 123 \overline{) 380} \\ 369 \\ \hline 12608 \overline{) 110000} \\ 100864 \\ \hline \text{rem. } \cdot 00009136 \end{array}$

Exer.

$$\begin{array}{r}
 5. \quad .0793 \\
 \quad \quad 4 \quad \overline{[.2816 \text{ Ans.}]} \\
 48 \overline{)393} \\
 \quad 384 \\
 \quad \quad 561 \overline{)900} \\
 \quad \quad \quad 561 \\
 5626 \overline{)33900} \\
 \quad \quad 33756 \\
 \text{rem. } .00000144
 \end{array}$$

$$\begin{array}{r}
 6. \quad .0005,55,55,55 \\
 \quad \quad 4 \quad \overline{[.02357 \text{ Ans.}]} \\
 43 \overline{)155} \\
 \quad 129 \\
 465 \overline{)2655} \\
 \quad 2325 \\
 4707 \overline{)33055} \\
 \quad 32949 \\
 \text{rem. } .0000000106
 \end{array}$$

Exer.

$$\begin{array}{r}
 7. \quad 29 \cdot 36, 36, 36, 36, 36 \\
 \quad \quad 25 \quad \overline{[5 \cdot 41882 \text{ Ans.}]} \\
 104 \overline{)436} \\
 \quad 416 \\
 1081 \overline{)2036} \\
 \quad 1081 \\
 10828 \overline{)95536} \\
 \quad 86624 \\
 108368 \overline{)891236} \\
 \quad 866944 \\
 1083762 \overline{)2429236} \\
 \quad 2167524 \\
 \text{rem. } .0000261712
 \end{array}$$

$$\begin{array}{r}
 8. \quad \frac{3}{4} + \frac{1}{4} = \frac{4}{4} \text{ and } \frac{4}{4} - \frac{1}{4} = \frac{3}{4} \\
 \quad \text{and } \frac{3}{4} = .75; \text{ then} \\
 \quad .26, 11, 11, 11, 11 \\
 \quad \quad 25 \quad \overline{[.51099 \text{ Ans.}]} \\
 101 \overline{)111} \\
 \quad 101 \\
 10209 \overline{)101111} \\
 \quad 91881 \\
 102189 \overline{)923011} \\
 \quad 919701 \\
 \text{rem. } .0000003310
 \end{array}$$

EXTRACTION OF THIRD ROOT.

(Arithmetic, page 198.)

Exer.

9

812·000,000,000,000,000,000

729

[9·329363 Answ.

$$9^3 \times 3 = 243 \dots 83000$$

$$9 \times 3 \times 3 = 81 \dots$$

$$3^2 = 9 \dots$$

$$3 \times 25119 = 75357$$

7643000

$$93^3 \times 3 = 25947 \dots$$

$$2 \times 93 \times 3 = 558 \dots$$

$$2^2 = 4 \dots$$

$$2 \times 2600284 = 5200568$$

2442432000

$$932^3 \times 3 = 2605872 \dots$$

$$9 \times 932 \times 3 = 25164 \dots$$

$$9^2 = 81 \dots$$

$$9 \times 260838921 = 2347550289$$

94881711000

$$9329^3 \times 3 = 261090723 \dots$$

$$3 \times 9329 \times 3 = 83961 \dots$$

$$3^2 = 9 \dots$$

$$3 \times 26109911919 = 78329735757$$

16551975243000

$$93293^3 \times 3 = 26110751547 \dots$$

$$6 \times 93293 \times 3 = 1679274 \dots$$

$$6^2 = 36 \dots$$

$$6 \times 2611091947476 = 15666551684856$$

885423558144000

$$932936^3 \times 3 = 2611108740288 \dots$$

$$3 \times 932936 \times 3 = 8386424 \dots$$

$$3^2 = 9 \dots$$

$$3 \times 261110957993049 = 783332873979147$$

rem. ·000102090684164853

EXTRACTION OF THIRD ROOT.

Exer.

10.

$$\begin{array}{r}
 659\text{--}000\text{--}000\ 000 \\
 \underline{512} \qquad [8\text{--}702188 \text{ Ans.}] \\
 147000 \\
 8^3 \times 3 = 192 \dots \\
 7 \times 8 \times 3 = 168 \dots \\
 \underline{7^3 = 49} \\
 7 \times 20929 = 146503 \\
 \hline
 870^3 \times 3 = 2270700 \dots \\
 2 \times 870 \times 3 = 5220 \dots \\
 \underline{2^3 = 4} \qquad 497000000 \\
 2 \times 227122204 = 454244408 \\
 \hline
 8702^3 \times 3 = 227174412 \dots \\
 1 \times 8702 \times 3 = 26106 \dots \\
 \underline{1^3 = 1} \qquad 42755592000 \\
 1 \times 22717702261 = 22717702261 \\
 \hline
 87021^3 \times 3 = 22717963323 \dots \\
 8 \times 87021 \times 3 = 2088504 \dots \\
 \underline{8^3 = 64} \qquad 20037889739000 \\
 8 \times 2271817217404 = 18174537739232 \\
 \hline
 870218^3 \times 3 = 2271838102572 \dots \\
 8 \times 870218 \times 3 = 20885232 \dots \\
 \underline{8^3 = 64} \qquad 186335199968000 \\
 8 \times 227184019109584 = 1817472152876672 \\
 \hline
 \text{rem. } 000045879846891328
 \end{array}$$

11.

$$\begin{array}{r}
 29\text{--}219\text{--}000 \\
 \underline{27} \qquad [3\text{--}08 \text{ Ans.}] \\
 2219000 \\
 30^3 \times 3 = 2700 \dots \\
 8 \times 30 \times 3 = 720 \dots \\
 \underline{8^3 = 64} \\
 8 \times 277264 = 2218112 \\
 \hline
 \text{rem. } 000888
 \end{array}$$

EXTRACTION OF THIRD ROOT.

69

Exer.

12.

$$\begin{array}{r}
 3\cdot009,200 \\
 \overline{) 3\cdot009,200} \\
 \underline{1\cdot000} \quad [1\cdot44372 \text{ Answ.}] \\
 2009 \\
 \begin{array}{r}
 1^3 \times 3 = 3 \dots \\
 4 \times 1 \times 3 = 12 \dots \\
 4^3 = 16 \\
 \hline
 4 \times 436 = 1744 \\
 \hline
 265200 \\
 \begin{array}{r}
 14^3 \times 3 = 588 \dots \\
 4 \times 14 \times 3 = 168 \dots \\
 4^3 = 16 \\
 \hline
 4 \times 60496 = 241984 \\
 \hline
 23216000 \\
 \begin{array}{r}
 144^3 \times 3 = 62208 \dots \\
 3 \times 144 \times 3 = 1296 \dots \\
 3^3 = 9 \\
 \hline
 3 \times 6233769 = 18701307 \\
 \hline
 4514693000 \\
 \begin{array}{r}
 1443^3 \times 3 = 6246747 \dots \\
 7 \times 1443 \times 3 = 30303 \dots \\
 7^3 = 49 \\
 \hline
 7 \times 624977779 = 4374844453 \\
 \hline
 139848547 \\
 \begin{array}{r}
 14437^3 \times 3 = 625280907 \dots \\
 2 \times 14437 \times 3 = 86622 \dots \\
 7^3 = 4 \\
 \hline
 2 \times 62528956924 = 125057913848 \\
 \hline
 \text{rem. } \cdot 000014790633152
 \end{array}
 \end{array}
 \end{array}
 \end{array}$$

EXTRACTION OF THIRD ROOT.

Exer.

13.

·000,000,366,048,000

$$\begin{array}{r}
 343 \quad \overline{) \cdot 00715 \text{ Answ.}} \\
 \underline{23048} \\
 7^3 \times 3 = 147 \dots \\
 1 \times 7 \times 3 = 21 \dots \\
 1^3 = 1 \dots \\
 \underline{1 \times 14911 = 14911} \\
 8137000 \\
 71^3 \times 3 = 15123 \dots \\
 5 \times 71 \times 3 = 1065 \dots \\
 5^3 = 25 \dots \\
 \underline{5 \times 1522975 = 7614875} \\
 \text{rem. } \cdot 000000000522125
 \end{array}$$

14.

·007,645,373

$$\begin{array}{r}
 1 \quad \overline{) 6645} \quad \overline{) 197 \text{ Answ.}} \\
 \underline{6645} \\
 1^3 \times 3 = 3 \dots \\
 9 \times 1 \times 3 = 27 \dots \\
 9^3 = 81 \dots \\
 \underline{9 \times 651 = 5859} \\
 786373 \\
 19^3 \times 3 = 1083 \dots \\
 7 \times 19 \times 3 = 399 \dots \\
 7^3 = 49 \dots \\
 \underline{7 \times 112339 = 786373}
 \end{array}$$

15.

$$^3\sqrt{49\frac{1}{27}} = ^3\sqrt{1\frac{1}{3}} = 1\frac{1}{3}$$

$$\begin{array}{r}
 1331 \quad [11 = \text{root of numerator.}] \\
 1 \\
 \underline{1} \\
 331 \quad [^3\sqrt{27} = 3 = \text{root of denom.}] \\
 1^3 \times 3 = 3 \dots \\
 1 \times 1 \times 3 = 3 \dots \\
 1^3 = 1 \dots \\
 \underline{1 \times 331} \quad 331 \\
 \dots \therefore \sqrt[3]{49\frac{1}{27}} = 3\frac{1}{3} = 3\cdot6' \text{ Answ.}
 \end{array}$$

Exer.

16.

$$\sqrt[3]{125} \quad 5$$

$$2009 = \sqrt[3]{2009}$$

2009

1. [12.61808 root of denom.

1009

$$1^2 \times 3 = 3 \dots$$

$$2 \times 1 \times 3 = 6 \dots$$

$$2^2 = 4$$

$$2 \times 364 = 728$$

281000

$$12^2 \times 3 = 432 \dots$$

$$6 \times 12 \times 3 = 216 \dots$$

$$6^2 = 36$$

$$6 \times 45396 = 272376$$

8624000

$$126^2 \times 3 = 47628 \dots$$

$$1 \times 126 \times 3 = 378 \dots$$

$$1^2 = 1$$

$$1 \times 4766581 = 4766581$$

3857419000

$$1261^2 \times 3 = 4770363 \dots$$

$$8 \times 1261 \times 3 = 30264 \dots$$

$$8^2 = 64$$

$$8 \times 477339004 = 3818712032$$

38706968000000

$$126180^2 \times 3 = 47764177200 \dots$$

$$8 \times 126180 \times 3 = 3028320 \dots$$

$$8^2 = 64$$

$$8 \times 4776448003264 = 38211584026112$$

rem. 000495383973888

 \therefore result = 396256 Answ.

EXTRACTION OF THIRD ROOT.

Exer.

17.

Nearest root=10.

$$10^3=1000$$

$$\begin{array}{r} 1000 \quad 987 \\ \underline{2} \quad \underline{2} \\ 2000 \quad 1974 \\ \underline{987} \quad \underline{1000} \end{array}$$

$$2987 : 2974 :: 10 : 9.956478 \text{ Answ.}$$

18.

Assumed root=6.

$$\begin{array}{r} 6^3=216 \quad 284 \\ \underline{2} \quad \underline{2} \\ 432 \quad 568 \\ \underline{284} \quad \underline{216} \\ 716 \quad : \quad 784 :: 6 : 6.5 \end{array}$$

Again, assume 6.5;

$$\text{then } 6.5^3=274.625 \quad 284$$

$$\begin{array}{r} \underline{2} \quad \underline{2} \\ 549.25 \quad 568 \\ \underline{284} \quad \underline{274.625} \end{array}$$

$$833.25 : 842.625 :: 6.5 : 6.57313 \text{ Ans.}$$

19.

$$\begin{array}{r} 0 \quad 0 \quad 1.685159 \\ \underline{1} \quad \underline{1} \quad \underline{1} \quad [1.19 \text{ Ans.}] \\ 1 \quad 1 \quad 685 \\ \underline{1} \quad \underline{2} \quad \underline{331} \\ 2 \quad 300 \quad 354159 \\ \underline{1} \quad \underline{31} \quad \underline{354159} \\ 30 \quad 331 \\ \underline{1} \quad \underline{32} \\ 31 \quad 36300 \\ \underline{1} \quad \underline{3051} \\ 32 \quad 39351 \\ \underline{1} \\ 330 \\ \underline{9} \\ 339 \end{array}$$

Exer.

20. 0 0 ·000,000,592,704

8	64	512	[·0084 Ans.
<u>8</u>	<u>64</u>	<u>80704</u>	
8	128	80704	
<u>16</u>	<u>19200</u>	<u> </u>	
8	976		
<u>240</u>	<u>20176</u>		
4			
<u>244</u>			

21. 0 0 ·000,001,003,003,001

1	1	1	[·01001 Ans.
<u>1</u>	<u>1</u>	<u>3003001</u>	
1	2	3003001	
<u>2</u>	<u>3000000</u>	<u> </u>	
1	3001		
<u>3000</u>	<u>3003001</u>		
1			
<u>3001</u>			

22. $\sqrt[3]{27} = \frac{3}{3375} = \sqrt[3]{3375}$

0	0	3,375	[15, root of denominator.
1	1	1	
<u>1</u>	<u>1</u>	<u>2375</u>	
1	2	2375	
<u>2</u>	<u>300</u>	<u> </u>	
1	175		
<u>30</u>	<u>475</u>		
5			
<u>35</u>			

$\therefore \frac{3}{\sqrt[3]{3375}} = \frac{3}{15} = \frac{1}{5}$ Ans.

Otherwise, reduce $\frac{3}{3375}$ to its lowest terms, and it becomes $\frac{1}{1125}$, and $\sqrt[3]{\frac{1}{1125}} = \frac{1}{15}$, as before.

74

Ex.

EXTRACTION OF THIRD ROOT.

23.

$\sqrt[3]{11} = .63'$

0	0	-636363	
8	64	512	[-86 + Answ.]
8	64	124363	
8	128	124056	
<u>16</u>	<u>19200</u>		
8	1476		
<u>240</u>	<u>20676</u>		
6			
<u>246</u>			

24.

$\sqrt[3]{.14} = .0000537824^{\frac{1}{3}}$

0	0	.000,053,782,400	
3	9	27	[-0377 Answ.]
3	9	26782	
3	18	23653	
<u>6</u>	<u>2700</u>	<u>3129400</u>	
3	679	2929633	
<u>90</u>	<u>3379</u>		
7	728		
<u>97</u>	<u>410700</u>		
7	7819		
<u>104</u>	<u>418519</u>		
7			
<u>1110</u>			
7			
<u>1117</u>			

Exer.

$$25. \quad 14.69693385^{\frac{2}{3}} = 214.717793591^{\frac{1}{3}}$$

0	0	214.717,793,591	
5	25	125	{ 5.988, &c., or 6 nearly. Ans.
5	25	89717	
5	50	80379	
10	7500	9338793	
5	1431	8468192	
150	8931	870601591	
9	1512	859398272	
159	1044300		
9	14224		
168	1058524		
9	14288		
1770	107281200		
8	143584		
1778	107424784		
8			
1786			
8			
17940			
8			
17948			

26.

$$^3\sqrt{1} = 1.$$

$$^3\sqrt{.001} = .1$$

$$^3\sqrt{1.030,301} = 1.01$$

$$\begin{array}{r}
 10^3 \times 3 = 300 \dots \\
 1 \times 10 \times 3 = 30 \dots \\
 1^3 = 1 \dots \\
 \hline
 1 \times 30301 = 30301
 \end{array}$$

$$\therefore 1^{\frac{1}{3}} + .001^{\frac{1}{3}} - 1.030301^{\frac{1}{3}} = 1 + .1 - 1.01 = .09 \text{ Ans.}$$

ROOTS IN GENERAL.

(Arithmetic, page 205.)

Exer. 27.

$$\begin{array}{r}
 0 \\
 1 \\
 1 \\
 1 \\
 5 \\
 6 \\
 15 \\
 2100000 \\
 1427613 \\
 3527613 \\
 \hline
 0 \\
 1 \\
 1 \\
 4 \\
 5 \\
 10 \\
 15 \\
 20 \\
 350000 \\
 125871 \\
 475871 \\
 \hline
 0 \\
 1 \\
 1 \\
 3 \\
 4 \\
 6 \\
 10 \\
 10 \\
 20 \\
 15 \\
 35000 \\
 6957 \\
 41957 \\
 \hline
 0 \\
 1 \\
 1 \\
 2 \\
 3 \\
 3 \\
 6 \\
 4 \\
 10 \\
 5 \\
 15 \\
 6 \\
 2100 \\
 219 \\
 2319 \\
 \hline
 0 \\
 1 \\
 1 \\
 1 \\
 2 \\
 1 \\
 3 \\
 1 \\
 4 \\
 1 \\
 5 \\
 1 \\
 6 \\
 1 \\
 70 \\
 3 \\
 73
 \end{array}$$

6-2748517
1
52748517
52748517

0
1
1
6
7000000
10582839
17582839

1.3 Ans.

Exer. 28.				Exer. 29.			
0	0	0	1116·79136, 18807	0	0	0	5,37824
4	16	64	1024	1	1	1	1
4	16	64	<u>927913618807</u>	1	1	1	<u>437824</u>
4	32	192	1024	1	2	3	<u>437824</u>
8	48	256	<u>128000000000</u>	2	3	4	<u>50000</u>
4	48	384	<u>4559088401</u>	1	3	6	<u>59456</u> [14Ans.
12	96	<u>640000000</u>	<u>132559088401</u>	3	6	<u>10000</u>	<u>109456</u>
4	64	11298343		1	4	4864	
16	<u>1600000</u>	<u>651298343</u>		4	<u>1000</u>	<u>14864</u>	
4	14049			1	216		
<u>2000</u>	<u>1614049</u>			50	<u>1216</u>		
7				4			
<u>2007</u>				54			

Exer. 30.

$\left(\frac{5}{9}\right)^{\frac{1}{11}} = \sqrt[11]{5}$; assume 1 as the root, then $P=5'$, $n=11$, $A=1$, $r=1$; and

$$\frac{12 \times 1 + 10 \times 5'}{12 + 5 \cdot 55555} : \frac{12 \times 5' + 10 \times 1}{6 \cdot 66636 + 10} :: 1 : .94.$$

Again assume .94. $\therefore .94^{11} = .506298207249$

then $12 \times .506298207249 + 10 \times 5' : 12 \times 5' + 10 \times .506298207249 :: .94 : 94796$ Ans.

ROOTS IN GENERAL.

Exer. 33.

$$3.75^{\frac{1}{2}} = 52.734375^{\frac{1}{2}}$$

0	0	0	0
1	1	1	1
1	1	1	1
1	2	3	4
2	3	4	5
1	3	6	10
3	6	10	15
1	4	10	20
4	10	20	350000
1	5	15	374311
5	15	35000	724311
1	6	18473	532434
6	2100	53473	1256745
1	539	22589	721770
70	2639	76062	1978515
7	588	27048	944720
77	3227	103110	29232350000
7	637	31850	1053836736
84	3864	134960	30286186736
7	686	36995	1076203164
91	4550	171955000	31362389840
7	735	3684456	1098830400
98	5285	175639456	32461220240
7	784	3727728	1121719920
105	606900	179367184	335829401600000
7	7176	3771216	381884616976
112	614076	183138400	336211286216976
7	7212	3814920	
1190	621288	186953320	
6	7248	3858840	
1196	628536	190812160000	
6	7284	130148488	
1202	635820	190942308488	
6	7320		
1208	643140		
6	7356		
1214	65049600		
6	24644		
1220	65074244		
6			
1226			

N.B.—Read Exercise 38 across the two pages.

ROOTS IN GENERAL.

79

Exer. 33—continued.

0	0	52·7343750, [1·762 Answ.
1	1	1
1	1	517343750
5	6	400338673
6	7000000	1170050770000000
15	50191239	1127624903534976
2100000	57191239	424258664650240000000
5070177	111771744	416502894841112135808
7170177	168962983000000	
8797215	18974500922496	
15967392	187937483922496	
13839605	20103546956736	
2980699700000	208041030879232000000	
181717120416	710416541324067904	
3162416820416	208751447420556067904	
188174339040		
3350591159456		
194767321440		
354535848089600000		
672422572433952		
355208270662033952		

Exer. 32.

0	0	0	0	0	24,137569
1	1	1	1	1	1 [17 Answ.
1	2	1	1	1	23137569
1	3	3	4	5	23137569
2	3	4	5	600000	
1	6	6	10	2705367	
3	4	10	150000	3305367	
1	10	10	236481		
4	5	20000	386481		
1	60	13783			
5	1500	33783			
1	469				
60	1969				
7					
67					

Exer. 34.

$$.045^{\frac{1}{2}} = .000000098303765625^{\frac{1}{2}}$$

0	0	0	0	.000000083,03765,62500
2	4	8	16	32
2	4	8	16	5103765
2	8	24	64	4762624
4	12	32	800000	34114162500
2	12	48	390656	33735187232
6	24	80000	1190656	3789752680000000000
2	16	17664	468224	34303.9042373136032
8	4000	97664	1658880000	
2	416	19392	278793616	
100	4416	117056	16867593616	
4	432	21184	281116864	
104	4848	138240000	1714871048000000000	
4	448	1156808	283473186568016	
108	5296	139396808	1715154521186568016	
4	464	1161624		
112	576000	140558432		
4	2404	1166448		
116	578404	1417248800000000		
4	2408	11713284008		
1200	580812	141736593284008		
2	2412			
1202	583224			
2	2416			
1204	5856400000			
2	242004			
1206	5856642004			
2				
1208				
2				
121000				
2				
121002				

Exer. 35.

$$\begin{aligned}\sqrt{27} &= \sqrt{9 \times 3} = 3\sqrt{3} \\ \sqrt{24} &= \sqrt{4 \times 6} = 2\sqrt{6} \\ \text{Sum, } 3\sqrt{3} + 2\sqrt{2} &\text{ Ans.}\end{aligned}$$

Exer. 36.

$$\begin{aligned}7.84 \times 25 &= 1.96 \quad [14 \text{ Ans.}] \\ &\quad \underline{1} \\ 24) &\quad 96 \\ &\quad \underline{96} \\ &\quad 0\end{aligned}$$

ROOTS IN GENERAL.

Exer. 37.

0	0	2	[1·259921 Answ.
1	1	1	
1	1	1000	
1	2	728	
2	300	272000	
1	64	225125	
30	364	46875000	
2	68	42491979	
32	43200	4383021000	
2	1825	4282778799	
34	45025	100242201000	
2	1850	95242392488	
360	4687500	4999808512000	
5	33831	4762198998961	
365	4721331		
5	33912		
370	475524300		
5	340011		
3750	475864311		
9	340092		
3759	47620440300		
9	755944		
3768	47621196244		
9	755948		
37770	4762195219200		
9	3779761		
37779	4762198998961		
9			
37788			
9			
377970			
2			
377972			
2			
377974			
2			
3779760			
1			
3779761			

Exer. 38.

0	3	[1·7320508 Ans.
1	1	
1	200	
1	189	
20	1100	
7	1029	
27	7100	
7	6924	
340	1760000	
• 3	1732025	
343	279750000	
3	277128064	
3460		
2		
3462		
2		
346400		
5		
346405		
5		
34641000		
8		
34641000		
8		
34641008		

Exer. 39.

$$12\sqrt{2} = \sqrt{144 \times 2} = \sqrt{288}$$

0	2,88	{	root.	0	0	4	[1·587401
1	1		16·970562	1	1	1	72
<u>1</u>	188			<u>1</u>	<u>1</u>	3000	114·292872
1	156			1	2	2375	Ans.
20	3200			2	300	625000	
6	2961			1	175	569312	
26	23900			30	475	55688000	
6	23709			5	200	52657003	
320	1910000			35	67500	3030997000	
9	1697025			5	3664	3023044624	
329	21297500			40	71164	7952376000000	
9	20364636			5	3728	7559521042201	
3380	93286400			450	7489200		
7	67882244			8	33229		
3387				458	7522429		
7				8	33278		
339400				466	755570700		
5				8	190456		
339405				4740	755761156		
5				7	190472		
3394100				4747	7559516280000		
6				7	4762201		
3394106				4754	7559521042201		
6				7			
33941120				47610			
2				4			
33941122				47614			
				4			
				47618			
				4			
				4762200			
				4			
				4762201			

Exer. 40.		$(5+7)^{\frac{1}{2}}=12^{\frac{1}{2}}$							
0	0	12	{	root.	0	0	5	{	root.
2	4	8	{ $2 \cdot 289428 = 12^{\frac{1}{2}}$		1	1	1	{ $1 \cdot 709975 = 5^{\frac{1}{2}}$	
2	4	4000			1	1	4000		
2	8	2648			1	2	3913		
4	1200	1352000			2	300	87000000		
2	124	1204352			1	259	78443829		
60	1324	147648000			30	559	8556171000		
2	128	140911569			7	308	7889992299		
62	145200	6736431000			37	8670000	666178701000		
2	5344	6288523984			7	45981	614014317973		
64	150544	447907016000			44	8715981	52164383627000		
2	5408	314483888888			7	46062	43860089261375		
660	15595200	133423127112000		5100	876204300				
8	61641	125795094042752		9	461511				
668	15656841			5109	876665811				
8	61722			9	461592				
676	1571856300			5118	87712740300				
8	274696			9	3590839				
6840	1572130996			51270	87716331139				
9	274712			9	3590888				
6849	157240570800			51279	8771992202700				
9	1373644			9	25649575				
6858	157241944444			51288	8772017852275				
9	1373648			9					
68670	15724331809200			512970					
4	54946144			7					
68674	15724386755344			512977					
4				7					
68678				512984					
4				7					
686820				5129910					
2				5					
686822				5129915					
2									
686824									
2									
6868260									
8									
6868268									

$$15^{\frac{3}{2}} = 15^{\frac{1}{2}} = 15^{\frac{3}{2}}; \text{ then } 15 \times 15 \times 15 = 3375 \text{ Answ.}$$

$$\left(\frac{18}{2}\right)^3 = 9^3; \text{ then } 9 \times 9 \times 9 = 729.$$

$$\frac{18^3}{2} = \frac{9^3 \times 2^3}{2} = 9^3 \times 2^2 = 729 \times 4 = 2916; \text{ then } 2916 - 729 = 2187 \text{ Answ.}$$

N.B.—Read Exercise 40 across the two pages.

Exer. 40—continued.

0	0	7			
1	1	1			$1 \cdot 912931 = 7^{\frac{1}{2}}$
1	1	6000	}	root,	add $1 \cdot 709975 = 5^{\frac{1}{2}}$
1	2	5859		$1 \cdot 912931 = 7^{\frac{1}{2}}$	from $3 \cdot 622906 = (7^{\frac{1}{2}} + 5^{\frac{1}{2}})$
2	300	141000		take	$2 \cdot 289428 = 12^{\frac{1}{2}}$
1	351	108871		Ans.	$1 \cdot 333478 = (7^{\frac{1}{2}} + 5^{\frac{1}{2}}) \sim 12^{\frac{1}{2}}$
30	651	32129000			
9	432	21911528			
39	108300	10217472000		$7^{\frac{1}{2}} = (7^{\frac{1}{2}})^2 = 1 \cdot 912931^2$	
9	571	9875155689			1 912931
48	108871	342316311000			1912931
9	572	329331941757			1721638
570	10944300	12984369243000			19129
1	11464	10977909298491			3826
571	10955764				1722
1	11468				57
572	1096723200				2
1	516821				$3 \cdot 659305 = 7^{\frac{1}{2}}$
5730	1097239521				5
2	516402				$18 \cdot 296525 = (5 \times 7^{\frac{1}{2}})$
5732	109775592300				
2	1721619				
5734	109777313919				15 [3·872983 = $\sqrt{15}$.
2	1721628				9
57360	10977903554700				68)600
9	5738791				544
57369	10977909293491				767)5600
9					5369
57378					7742)23100
9					15484
573870					77449)761600
3					697041
573873					774588)6455900
3					6196704
573876					7745963)25919600
3					23237889
5738790					
1					
5738791					
0					
3					
3					
3					
60					
8					
68					

From $18 \cdot 296525 = (5 \times 7^{\frac{1}{2}})$
take $3 \cdot 872983 = \sqrt{15}$

$14 \cdot 423542 = [(5 \times 7^{\frac{1}{2}}) - \sqrt{15}]$

$[3 \cdot 8 \text{ nearly} - [(5 \times 7^{\frac{1}{2}}) - \sqrt{15}]^{\frac{1}{2}}]$ Ans.

EQUI-DIFFERENT SERIES.

Exer.

(Arithmetic, page 207.)

1. $3 + 2(65-1) \quad 3 + 2 \times 64 = 131 = 65\text{th term.} \quad \text{Answ.}$

2. $10 - .01 \times 999 = 10 - 9.99 = .01. \quad \text{Answ.}$

3. $5 + 2 \times 23 = 51s. = £2 \quad 11 \quad 0 = 24\text{th term;}$

$$\frac{5 + 51 \times 24}{2} = 56 \times 12 = 672s. = £33 \quad 12 \quad 0, \text{ sum. Ans.}$$

4. $12 - (.75 \times 7) = 12 - 5.25 = 6.75 = \text{less extreme;}$

$$\frac{12 + 6.75 \times 8}{2} = 18.75 \times 4 = 75 = \text{sum.} \quad \text{Answ.}$$

5. $l = 2, g = 300, n = 150, d = 2;$

$$\frac{2 + 300 \times 150}{2} = 302 \times 75 = 22650 \text{ yards, sum;}$$

$$220)22650(102 \text{ furlongs.}$$

22

65

44

210

fur.	yds.
8)102	210

12	6	210	Answ.
----	---	-----	-------

mls.	fur.	yds.
------	------	------

6. $1 + 2 \times 99 = 199 = \text{greater term of odd numbers,}$

$$1 + 99 \times 50 = 10000 = \text{sum of odd numbers,}$$

$$2 + 2 \times 99 = 200 = \text{greater term of even numbers,}$$

$$2 + 200 \times 50 = 10100 = \text{sum of even numbers;}$$

$$\text{then } 10100 - 10000 = 100 = \text{difference.} \quad \text{Answ.}$$

7. $1 + \frac{24 - 10\frac{1}{2}}{1\frac{1}{2}} = 1 + \frac{13\frac{1}{2}}{1\frac{1}{2}} = 1 + \frac{27}{3} = 10, \text{ number of days,}$

$$(24 + 10\frac{1}{2}) \times 5 = 172\frac{1}{2} \text{ miles} = \text{sum.} \quad \text{Answ.}$$

8.
$$\frac{[(2 \times 20 - 20)^2 + 8 \times 20 \times 560]^{\frac{1}{2}} - (2 \times 20 - 20)}{2 \times 20} =$$

$$\frac{(20^2 + 89600)^{\frac{1}{2}} - 20}{40} = \frac{(400 + 89600)^{\frac{1}{2}} - 20}{40} =$$

$$\sqrt{\frac{90000 - 20}{40}} = \frac{300 - 20}{40} = \frac{280}{40} = 7 \text{ yrs.} = \text{number of terms.}$$

[Answ.]

Exer.

9. $\frac{2(25 \times 63 - 825)}{25 \times 24} = \frac{1575 - 825}{25 \times 12} = \frac{750}{300} = 2\frac{1}{2} = \text{com. dif. Answ.}$
10. $620 - 9 \times 60 = 620 - 540 = £80 = \text{last term. Answ.}$
11. Because in such a series, by throwing out all the even numbers, the middle term must always be equal to the number of terms; and because the middle term is likewise equal to half the sum of the extremes; therefore the middle term multiplied by itself in such a case will correspond with the rule. *Art. 278.*
12. $.02 \times 50 = 1 = \text{greater,}$
 $\frac{51 \times 1}{2} = 25.5 = \text{sum. Answ.}$

EQUI-RATIONAL SERIES.

(Arithmetic, page 314.)

1. $a=2, r=3.5, n=5$, to find z, s , and mean;
 $3.5 \times 3.5 = 12.25 \times 12.25 = 150.0625 = r^4$

$$\begin{array}{r}
 2=a \\
 300.125=\text{greater term,} \\
 3.5 \\
 1500625 \quad 12.25=r^2 \\
 900375 \quad 2=a \\
 1050.4375 \quad 24.5 = \text{mean.} \\
 2. \\
 2.5)1048.4375 \\
 \underline{100} \quad [419.375 \text{ sum.} \\
 48 \\
 25 \\
 \underline{234} \\
 225 \\
 \underline{93} \\
 75 \\
 \underline{187} \\
 175 \\
 \underline{125} \\
 125 \\
 \underline{\quad}
 \end{array}$$

Exer.

- 2.
- $z=12$
- ,
- $r=\frac{1}{2}$
- ,
- $n=10$
- , to find
- a
- ;

$$\left(\frac{1}{2}\right)^0 = \frac{1}{2}, \quad \left(\frac{1}{2}\right)^1 = \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^2; \text{ then } \frac{1}{2} \times 12 = \frac{1}{2} \times 12 = \frac{1}{2} \times 12 = 10^{\text{th}} \text{ term.}$$

- 3.
- $r=3$
- ,
- $n=7$
- ,
- $z=\pounds 109350$
- 729)109350 (£150=first term. Ans.

$$\begin{array}{r} 3^0=27 \\ 27^0=729=3^0=r \end{array} \quad \begin{array}{r} 729 \\ 3645 \\ 3645 \end{array}$$

- 4.
- $z=.93$
- ,
- $r=.01$
- ,
- $n=\text{infinity}$
- ;
-
- $.93 \div (1-.01) = .99$
- sum. Ans.

- 5.
- $a=2$
- ,
- $r=2$
- ,
- $n=15$
- $5\frac{1}{2}$
-)65534
-
- $2^0=8$
- ,
- $8^0=512=2^0$
- 11)131068
-
- $64=2^0$
- 40)11915.3 half yds.
-
- $32768=2^{15}$
- 8)297.35.3 h. yds.
-
- $2=a$
- 37.1.35.1
- $\frac{1}{2}$
- Ans.
-
- from 65536
-
- take 2=a
-
- mls. fur. per. yds.

$$(2-1)=1)65534$$

65534 yards=sum.

- 6.
- $a=\frac{1}{8}$
- ,
- $z=8192$
- ,
- $s=10922\frac{5}{8}$
- ,
-
- $10922\frac{5}{8} - \frac{1}{8} = 10922\frac{4}{8} = 10922.5$
- ,
-
- $10922\frac{5}{8} - 8192 = 2730\frac{5}{8} = 2730.625$
- ,
-
- $2730.625)10922.500(4=r$
-
- 10922.500

$$1 + \frac{\log .8192 - \log .125}{\log 4} = 1 + \frac{3.91339 - 1.09691}{.60206}$$

$$\begin{array}{r} 3.91339 \\ 1.09691 \\ \hline .60206)4.81648(8 \\ 4.81648 \end{array}$$

then $1+8=9$ =number of terms. Ans.

8. Subtracting the terminate part, there remains
-
- .005, the greater term; then .1 being the ratio,
-
- $.005 \div .9 = .005555$
- = value of interminate part, and

$$\frac{72}{100} + \frac{5}{900} = \frac{648+5}{900} = \frac{653}{900} \text{ Ans.}$$

Exer.

7. $l=3$, $d=2$, $n=12$, to find s ,

$$4 \times (2 \times 3 + 11 \times 12) = 6 \times (6 + 22) = 6 \times 28 = 168 = \text{sum of equidifferent series.}$$

$a=3$, $r=2$, $n=12$, to find s ,

$$2^2=8 \times 8=64=2^6 \text{ and } 64 \times 64=4096=2^{12}$$

from 4096

take 1

$$(2-1)=1)4095$$

$$4095 \times 3 = 12285 = \text{sum of equi-rat. series.}$$

168

$$12117 = \text{difference. Ans.}$$

9. $x=1$, $r=\frac{1}{100}$, $n=\text{infinity}$;

Then $1 \div (1 - 0.01) = .99 = \frac{99}{100} = 1\frac{1}{100}$ furlongs = the distance moved by Achilles before he overtakes the tortoise; but the tortoise was 1 furlong in advance at starting; therefore $1\frac{1}{100} - 1 = \frac{1}{100}$ th of a furlong moved by the tortoise. Ans.

$$10. .21 \div .99 = \frac{21}{99} = \frac{7}{33} = \text{Ans.}$$

$$11. .1234567890 = z, .0000000001 = r;$$

$$\text{then } .1234567890 \div 9999999999 = \frac{1234567890}{9999999999} = \frac{123456789}{999999999} = \text{Ans.}$$

LOGARITHMS.

(Arithmetic, page 219.)

Exer. 1.

Log. 101=2.0043214	}	Ans.
— 743=2.8709888		
— 11=1.0413927		
— 100=2.		
— 2=0.3010301		
		12

Exer. 2.

$$7263 = 3 \times 3 \times 3 \times 269$$

$$\log \text{ of } 3^3 = 1.4313639$$

$$\text{— } 269 = 2.4297523$$

$$\text{— } 7263 = 3.8611162 \text{ Ans.}$$

$$96 = 2^5 \times 3$$

$$\log \text{ of } 2^5 = 1.50515$$

$$\text{— } 3 = 0.4771213$$

$$\text{— } 96 = 1.9822713 \text{ Ans.}$$

$$854 = 2 \times 7 \times 61$$

$$\log \text{ of } 2 = 0.301030$$

$$\text{— } 7 = 0.845098$$

$$\text{— } 61 = 1.7853298$$

$$\text{— } 854 = 2.9314578 \text{ Ans.}$$

$$6 = 2 \times 3$$

$$\log \text{ of } 2 = 0.301030$$

$$\text{— } 3 = 0.4771213$$

$$\text{— } 6 = 0.7781513 \text{ Ans.}$$

Exer. 3.

$$312 = 2^3 \times 3 \times 13$$

$$\log \text{ of } 2^3 = 0.903090$$

$$\text{— } 3 = 0.4771213$$

$$\text{— } 13 = 1.1139434$$

$$\text{— } 312 = 2.4941547 \text{ Ans.}$$

$$18 = 2 \times 3^2$$

$$\log \text{ of } 2 = 0.30103$$

$$\text{— } 3^2 = 0.9542426$$

$$\text{— } 18 = 1.2552726 \text{ Ans.}$$

$$2004 = 2^2 \times 3 \times 167$$

$$\log \text{ of } 2^2 = 0.60206$$

$$\text{— } 3 = 0.4771213$$

$$\text{— } 167 = 2.222716$$

$$\text{— } 2004 = 3.3018973 \text{ Ans.}$$

$$\log \text{ of } 1 = 0 \text{ Ans.}$$

$$19536 = 2^4 \times 3 \times 11 \times 37$$

$$\log \text{ of } 2^4 = 1.20412$$

$$\text{— } 3 = 0.4771213$$

$$\text{— } 11 = 1.0413927$$

$$\text{— } 37 = 1.5682017$$

$$\text{— } 19536 = 4.2908357$$

4. Searching the table, we find that the number lies between 157 and 163; compute the log. of 160; thus—

$$160 = 2^5 \times 5$$

$$\log \text{ of } 2^5 = 1.50515$$

$$\text{— } 5 = 0.69897$$

$$160 = 2.20412, \text{ which is the}$$

given log.; therefore 160 is the number required.

5. In the table the decimal part stands opposite 109, and as the index is 4, three ciphers are to be prefixed; therefore the number is .000109. Ans.

Exer.

6. $\cdot 8656961$ is not in the table, but lies between the logs. of 733 and 739; compute the log. of $736=2^3 \times 23$.

$$\log. \text{ of } 2^3 = 1.50515$$

$$\text{— } 23 = 1.3617278$$

$$\text{— } 736 = 2.8668778; \text{ this is too much.}$$

compute $\log. 734 = 2 \times 367$:

$$\log. \text{ of } 2 = .30103$$

$$\text{— } 367 = .5646661$$

$$\text{— } 734 = .8656961, \text{ corresponding}$$

to the decimal part of the given log.; and as the index is 1, there must be two places of integers; therefore the number is 73.4. *Ans.*

Number to $\log. 3 = 1000$. *Ans.*

$\cdot 7331973$ corresponds to 541; and as the index is 1, there must be a cipher prefixed; thus—

$$\cdot 0541 \text{ *Ans.*}$$

Number to $\log. 1 = 10$, *Ans.*

Number corresponding to $\cdot 7242759 = 53$; and 2 being the index, there are three places; thus—530, *Ans.*

$\log. 2 = 100$, $\log. 1 = 10$, $\log. 1 = \bar{1}$, $\log. \bar{2} = .01$.
[*Ans.*]

MULTIPLICATION BY LOGARITHMS.

(*Arithmetic*, page 224.)

Exer.

$$\begin{array}{r} 1. \quad 4.1 \times 1.79 \\ \log. \text{ of } 4.1 = 0.6127839 \\ \text{— } 1.79 = 0.2528530 \\ \hline \text{Ans. } 7.339 = 0.8656369 \end{array}$$

$$\begin{array}{r} 2. \quad \log. \text{ of } 56 = 1.748188 \\ \text{— } .5 = \bar{1}.698970 \\ \text{— } .003 = \bar{3}.4771213 \\ \hline \text{Ans. } .0084 = 2.9242793 \end{array}$$

Exer.

$$\begin{array}{r} 3. \quad \log. \text{ of } 213 = 2.32838 \\ \text{— } .09 = \bar{2}.954243 \\ \text{— } \frac{2}{3} = \bar{1}.60206 \\ \hline \text{Ans. } 7.668 = 0.884683 \end{array}$$

$$\begin{array}{r} 4. \quad \log. \text{ of } 7.5 = 0.875061 \\ \text{index } 3 \\ \hline \text{Ans. } 421.874 = 2.62588^3 \end{array}$$

92 DIVISION, EVOLUTION, ETC., BY LOGARITHMS.

Exer.

$$5. \log. \text{ of } .007 = \bar{3}.845098$$

index 2

$$\text{Ans. } .000049 = \bar{5}.690196$$

$$6. \log. \text{ of } 29.1 = 1.463893$$

index 3

$$\text{Ans. } 24642.159 = 4.391679$$

$$7. \log. \text{ of } 2.05 = 0.311754$$

index 7

$$\text{Ans. } 152.152 = 2.182278$$

$$8. \log. \text{ of } .05 = \bar{2}.69897$$

index 6

$$.000000015625 = \bar{8}.19382$$

Ans.]

$$11. \quad \ddagger (= .9166') = \bar{1}.962180$$

$$\quad \ddagger (= .4') = \bar{1}.647774$$

$$(\ddagger)^2 = .8311 = \bar{2}.917022$$

$$.00826446 = \bar{3}.917022$$

$$12. \quad \log. .11 = \bar{1}.041393$$

index 13

$$.000000000000345227 = \bar{13}.538109$$

DIVISION, EVOLUTION, &c.

(Arithmetic, page 225.)

Exer.

$$1. \log. 217.2 = 2.336860$$

$$\quad - 26.53 = 1.423737$$

$$\text{Ans. } 8.187 = 0.913123$$

$$2. \log. .0685 = \bar{2}.835691$$

$$\quad - .0093 = 3.968483$$

$$\text{Ans. } 7.3656 = 0.867208$$

Exer.

$$3. \log. 5 = 0.698970$$

$$\quad - 13.4 = 1.127105$$

$$\text{Ans. } .37313 = \bar{1}.571865$$

$$4. \log. .29 = \bar{1}.462398$$

$$\quad - 156 = 2.193125$$

$$\text{Ans. } .001859 = \bar{3}.269273$$

Exer.

$$5. \log. 121^{\frac{1}{2}} = 2.085914$$

$$\quad \quad \quad \underline{53^{\frac{1}{2}} = 1.730782}$$

$$\text{Ans. } 2.2653 = 0.355182$$

$$6. \log. 236 = 2.372912$$

index 3)

$$\text{Ans. } 6.1797 = 0.790970$$

$$7. \log. 7.263^{\frac{1}{2}} = 0.861116$$

index 5)

$$\text{Ans. } 1.486 = 0.172223$$

$$8. \log. .00826 = \bar{3}.91698$$

index 6)

$$\text{Ans. } .4496 = \bar{1}.65283$$

$$9. \log. 363 = 2.559907$$

2

$$\quad \quad \quad \underline{5) 5.119814}$$

$$\text{Ans. } 10.56 = 1.023962$$

Exer.

$$10. \log. \text{ of } 6.2 = 0.792392$$

4

$$\quad \quad \quad \underline{5) 3.169568}$$

$$\quad \quad \quad \underline{6.2^{\frac{1}{4}} = 0.633913}$$

$$\log. \text{ of } .09 = \bar{2}.954243$$

11

$$\quad \quad \quad \underline{12) 12.496673}$$

$$\quad \quad \quad \underline{.09^{\frac{11}{12}} = \bar{1}.041389}$$

again—

$$\log. \text{ of } .09 = \bar{2}.954243$$

12

$$\quad \quad \quad \underline{11) 13.450916}$$

$$\log. \text{ of } .09^{\frac{11}{12}} = \bar{2}.859174$$

$$\quad \quad \quad \underline{.09^{\frac{11}{12}} = \bar{1}.041389}$$

$$\quad \quad \quad \underline{3.900563}$$

$$\text{from } 0.633913$$

$$\text{take } 3.900563$$

$$\text{Ans. } 541.2 = 2.733350$$

$$11. \text{ Log. of } 734 = 2.865696 \div 5.34 = (\text{index}) = .536647$$

$$= \log. \text{ of } 734^{\frac{1}{5.34}}.$$

$$\text{Log. of } 2.9 = 0.462398 \times 3.2 = 1.4796736 \div .56 =$$

$$2.642274 = \log. \text{ of } 2.9^{\frac{3.2}{.56}}$$

$$\text{from } 0.536647 = \log. \text{ of } 734^{\frac{1}{5.34}}$$

$$\text{take } 2.642274 = \log. \text{ of } 2.9^{\frac{3.2}{.56}}$$

$$\underline{3.894373} = \log. \text{ of } .007841 \text{ Ans.}$$

$$12. \text{ Log. of } 10 = 1.000000 \times .3010300 (\text{index}) = .30103$$

$$\text{then } 1) .30103$$

$$\underline{.30103} = \log. \text{ of } 2.0000000200 \text{ Ans.}$$

PROPORTION BY LOGARITHMS.

(Arithmetic, page 226.)

Exer.

13. $1.3 : .0104 :: 2.375$

$$\log. .0104 = \bar{2}.017033$$

$$\text{— } 2.375 = 0.375664$$

$$\hline 2.392697$$

$$\text{— } 1.3 = 0.113943$$

$$\text{Ans. } .019 = \bar{2}.278754$$

14.

$$45^a : 12^{\frac{1}{3}} :: 136^{\frac{1}{3}}$$

$$\log. \text{ of } 12 = 1.079181$$

4

$$3)4.316724$$

$$1.438908 = \log. 12^{\frac{1}{3}}$$

$$\log. \text{ of } 45 = 1.653213$$

2

$$\hline 3.306426 = \log. 45^a$$

$$\log. 136 = 2.133539$$

2

$$5)4.267078$$

$$0.85345 = \log. 136^{\frac{1}{3}}$$

$$\log. 12^{\frac{1}{3}} = 1.438908$$

$$\text{— } 136^{\frac{1}{3}} = 0.85345$$

$$\hline 2.292323$$

$$\log. 45^a = 3.306426$$

$$\text{Ans. } .0968048 = \bar{2}.985897$$

Exer. 15.

$$7.3^a \times 15^{\frac{1}{3}} : .008^{\frac{1}{3}} \times 100 :: 24 :$$

$$\log. 7.3 = 0.863323$$

2

$$\text{Again— } \log. .008 = 3.903090$$

3)

$$\log. 7.3^a = 1.726646$$

$$\log. 15 = 1.176091$$

5)

$$\log. 15^{\frac{1}{3}} = 0.235218$$

$$\text{— } 7.3^a = 1.726646$$

$$\log. 15^{\frac{1}{3}} \times 7.3^a = 1.961864$$

$$\log. .008^{\frac{1}{3}} = \bar{1}.301030$$

$$\text{— } 100 = 2.000000$$

$$\log. .008^{\frac{1}{3}} \times 100 = 1.301030$$

$$\log. 24 = 1.38021$$

$$\hline 2.681241$$

$$\log. 15^{\frac{1}{3}} \times 7.3^a = 1.961864$$

$$\text{Ans. } 5.24055 = 0.719377$$

GENERAL EXERCISES IN LOGARITHMS.

(Arithmetic, page 226.)

Exer.

$$\begin{aligned}
 1. \quad & \cdot 192 = 2^3 \times \cdot 2^3 \times 3 \\
 & \log. 2^3 = (\log. 2, \times 3) = 0.90309 \\
 & \quad \text{— } 2^3 = (\log. \cdot 2) \times 4 = \bar{3}.90309 \\
 & \quad \text{— } 3 = \quad \quad \quad 0.4771213
 \end{aligned}$$

$$\text{Ans. } \log. \cdot 192 = \bar{1}.2833013$$

2. Answer.—Because $5 \times 2 = 10$, and their logs. added = log. 10; and $100 \div 10 = 10$, and their logs. subtracted = log. 10.

$$\begin{aligned}
 3. \quad & \log. 1 = 0.000000 \\
 & \log. \cdot 1239 = \bar{1}.093071 \\
 & \log. 8.071 = 0.906929
 \end{aligned}$$

$$4. \quad \log. 9 = 0.9542426$$

$$\begin{array}{r}
 11 \\
 \hline
 10.4966686
 \end{array}$$

Then, 1 being the log. of 2 places of figures, 2 the log. of 3 places, &c., 10 must be the log. of 11 places.

$$\begin{array}{r}
 \log. 137 = 2.1367206 \\
 20 \\
 \hline
 42.734412
 \end{array}$$

log. of 43 places of figures. Ans.

$$\begin{array}{r}
 \log. \text{ of } 64 = 1.80618 \\
 4) \\
 \hline
 \log. 2.828428 = 0.451545
 \end{array}$$

$$6. \quad n = 1 + \frac{(\log. 3072 =) 3.487421 - (\log. 3 =) 0.477121}{(\log. 2 =) 0.30103}$$

that is—

$$\begin{array}{r}
 3.487421 - 0.477121 \\
 \hline
 0.30103
 \end{array} = 10$$

then $10 + 1 = 11$ Ans.

$$7. \quad 189 = 3^3 \times 7, \text{ therefore}$$

$$(\log. 3) \times 3 + \log. 7 = \log. \text{ of } 189.$$

$$10.29 \quad 7 \times \cdot 7^3 \times 3, \text{ therefore}$$

$$(3 \times \log. 7) + \log. 3 = \log. 10.29. \quad \text{Ans.}$$

Exer.

$$9. \quad n = \frac{\log. [6041(2-1)+3] - \log. 3}{\log. 2.} = \frac{(\log. 6044 =) 3.781324 - 0.477121}{.30103} =$$

$$3.304203 \div .30103 = 10.97, \text{ \&c. } \text{Ans.}$$

$$10. \quad \log. 27.8 = 1.444045$$

$$14.440450$$

Then, as the index is 14, the number of places of integers is 15.

$$\log. .05 = \overline{2}.69897$$

$$20.48455$$

Therefore the first significant figure must occupy the twentieth place to the right of the decimal point, and there will therefore be 19 ciphers.

$$11. \quad \left. \begin{array}{l} 1.2304489 = \log. \text{ of } 17 \\ .4342945 = \log. \text{ of } 2.718 \text{ \&c.} = \text{base} \end{array} \right\} = 2.8332131$$

[Answ.]

[Answ.]

$$12. \quad .4342945) 3.0000000 (6.9077549 \text{ Ans.}$$

$$26057670$$

$$39423300$$

$$39086505$$

$$\text{\&c. \&c.}$$

$$13. \quad \log. 12 = 1.079181$$

$$11$$

$$\log. 12^{11} = 11.877991$$

As the index is 11, the corresponding number contains 12 integral places.

$$\log. 11 = 0.41393$$

$$12$$

$$\log. \text{ of } 11^{12} = 12.496716$$

Therefore there are 13 integral places in the corresponding number.

Exer.

14.

$$\log. 2 = 0.30103$$

2)

$$\log. 2^{\frac{1}{2}} = 0.150515$$

again—

$$\log. 3 = 0.477121 \div 3 = 0.1590403 = \log. 3^{\frac{1}{3}}$$

then—

from 0.1505150

take 0.1590403

$$5) \overline{1.9914747} = \log. \frac{2^{\frac{1}{2}}}{3^{\frac{1}{3}}}$$

$$\overline{1.9982949} = \log. \left(\frac{2^{\frac{1}{2}}}{3^{\frac{1}{3}}} \right)^{\frac{1}{5}}$$

$$\log. 7 = 0.845098$$

5)

$$\overline{0.1690196} = \log. \sqrt[5]{7}$$

$$\overline{1.9982949} = \log. \left(\frac{2^{\frac{1}{2}}}{3^{\frac{1}{3}}} \right)^{\frac{1}{5}}$$

$$\overline{0.1673145} = \log. 1.47 = \left(\frac{2^{\frac{1}{2}}}{3^{\frac{1}{3}}} \right)^{\frac{1}{5}} \times \sqrt[5]{7} \text{ Ans.}$$

$$\log. 2 = 0.30103$$

6

$$\log. 2^6 = 1.80618$$

$$\log. 25 = 1.39794$$

2

$$\log. 25^2 = 2.79588$$

$$\overline{1.80618}$$

$$\log. 2^6 \times 25^2 = 4.60206$$

$$\log. 4 = 0.60206$$

3

$$\log. 4^3 = 1.80618$$

$$\log. 10^3 = 2.00000$$

$$\log. 4^3 \times 10^3 = 3.80618$$

$$\text{from } \log. 2^6 \times 25^2 = 4.60206$$

$$\text{take } \log. 4^3 \times 10^3 = 3.80618$$

$$\text{Ans. } \log. 6.25 = 0.79588$$

15. $\frac{\log. 10 = 1.000000}{\log. 12 = 1.079181} = .9266286 = 1 \div \log. 12 \text{ or base.}$

[Ans.]

17.

$$21^{\frac{1}{3}} : ? :: 17^{\frac{1}{3}} : 15^{\frac{1}{3}}$$

$$17^{\frac{1}{3}} : 15^{\frac{1}{3}} :: 21^{\frac{1}{3}} : ?$$

$$\log. 17 = 1.230449$$

3)

$$\log. 17^{\frac{1}{3}} = 0.410149$$

$$\log. 15 = 1.176091$$

5)

$$\log. 15^{\frac{1}{3}} = 0.235218$$

$$\log. 21 = 1.322219$$

3

$$5) 3.966657$$

$$\log. 21^{\frac{1}{3}} = 0.793331$$

$$\log. 15^{\frac{1}{3}} = 0.235218 \text{ add}$$

$$1.028549$$

$$\log. 17^{\frac{1}{3}} = 0.410149 \text{ subtract}$$

$$\log. 4.15336 = 0.618400 \text{ Ans.}$$

18.

$$\log. 5 = .69897$$

$$\log. 1000 = 3.$$

$$\text{then, because } 5^l = 1000$$

$$l \times \log. 5 = \log. 1000$$

$$\text{therefore } l = \frac{\log. 1000}{\log. 5} = \frac{3.00000}{.69897} = 4.292029 \text{ Ans.}$$

COMPOUND INTEREST.

(Arithmetic, page 230.)

Exer.

$$\begin{array}{r}
 1. \quad 1.05 \\
 \quad 1.05 \\
 \quad \hline
 \quad 525 \\
 105 \\
 \hline
 1.1025 \\
 1.1025 \\
 \hline
 55125 \\
 22050 \\
 \hline
 121275 \\
 \hline
 1.21550625 = 1.05^4 \\
 \quad 100 \\
 \hline
 \pounds 121.550625 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 2. \quad \log. 1.03 = 0.012837 \\
 \quad \quad \quad 8 \\
 \quad \quad \quad \hline
 \quad \quad \quad 0.102696 \\
 \quad \quad \log. 50 = 1.698970 \\
 \quad \hline
 \log. 63.3385 = 1.801666 \\
 \quad \quad \quad \text{[Ans.]}
 \end{array}$$

$$\begin{array}{r}
 3. \quad .04^3 = 1.124864 \\
 \quad 276.375 = \pounds 276 \text{ } 7 \text{ } 6 \\
 \quad \hline
 \quad 5624320 \\
 \quad 7874058 \\
 \quad \hline
 \quad 3374592 \\
 \quad 6749184 \\
 \quad \hline
 \quad 7874048 \\
 \quad 2249728 \\
 \hline
 \pounds 310.88428810 \text{ Ans.}
 \end{array}$$

Exer.

$$\begin{array}{r}
 4. \quad \log. 1.06 = 0.025306 \\
 \quad \quad \quad 11 \\
 \quad \quad \quad \hline
 \quad \quad \quad 2) 0.278366 \\
 \quad \quad \quad \hline
 \quad \quad \quad 0.139183 \\
 \quad \quad \log. 10 = 1.000000 \\
 \quad \hline
 \log. \text{amt. } 13.778 = 1.139183 \\
 \quad \quad \quad \text{[Ans.]}
 \end{array}$$

$$\begin{array}{r}
 5. \quad 1.05^4 = 1.215506 \left\{ \begin{array}{l} \text{See} \\ \text{Solution} \\ \text{of No. 1.} \end{array} \right. \\
 \quad 1.05^3 = 1.1025 \\
 \quad \quad \quad 6077530 \\
 \quad \quad \quad 2431012 \\
 \quad \quad \quad \hline
 \quad \quad \quad 13370566 \\
 \quad \quad \quad \hline
 \quad \quad \quad 1.3400953650 \\
 \quad \quad \quad 1000 \\
 \hline
 \text{Ans. } 1340.095365
 \end{array}$$

$$\begin{array}{r}
 6. \quad \log. 1.03 = 0.012837 \\
 \quad \quad \quad 15 \\
 \quad \quad \quad \hline
 \quad \quad \quad 4) 0.192555 \\
 \quad \quad \quad \hline
 \quad \quad \quad 0.048138 \\
 \quad \quad \log. 5.5 = 0.740303 \\
 \quad \hline
 \log. \text{amt. } 6.144 = 0.788441 \\
 \quad \quad \quad \text{[Ans.]}
 \end{array}$$

$$\begin{array}{r}
 7. \quad \log. 1.05 = 0.021189 \\
 \quad \quad \quad 24 \\
 \quad \quad \quad \hline
 \quad \quad \quad 84756 \\
 \quad \quad \quad 42378 \\
 \quad \quad \quad \hline
 \quad \quad \quad 0.508536 \\
 \quad \quad \log. 20 = 1.30103 \\
 \quad \hline
 \log. \text{amt. } 64.502 = 1.809566 \\
 \quad \quad \quad \text{[Ans.]}
 \end{array}$$

Exer.

8. $\log. 1.03 = 0.0128372$

1000

$\log. 68738, \&c. = 12.8372$

As 12 is the index there must be 13 places of integers, of which the 5 to the left are 68738. Ans.

9. $1.48877 = £394.7046$

20

$s.14.092$

12

$d.1.104$

$£394 \ 14 \ 1 \text{ Ans.}$

Exer.

10. $1.48877 = £72.4619$

20

$s.9.238$

12

$d.2.856$

4

$f.3.424$

$£72 \ 9 \ 2\frac{3}{4} \text{ Ans.}$

11. $1.48877 = £7.8352$

20

$s.16.704$

12

$d.8.448$

4

$f.1.792$

$£7 \ 16 \ 8\frac{1}{4} \text{ Ans.}$

12.

$\log. 1.065 = 0.02735$

15

4) 0.41025

0.1025625 take

$\log. 1700 = 3.2304489 \text{ from}$

$\log. 1342.416 = 3.1278864$

20

$s.8.32$

12

$d.3.84$

4

$f.3.36$

$£1342 \ 8 \ 3\frac{3}{4} \text{ Ans.}$

Exer.
13.

$$\begin{array}{r} \log. 300 = 2.477121 \\ \log. 5 = 0.69897 \\ \hline 3.176091 \text{ from} \\ \log. 100 = 2.000000 \text{ take} \\ \hline 1.176091 \\ \log. 9 = 0.954243 \\ \hline \log. £135 = 2.130334 \end{array}$$

£135 = simple interest of £300 for 9 years at 5 per cent.

$$\begin{array}{r} \text{Again—} \quad \log. 1.05 = 0.021189 \\ \hline 9 \\ 0.190701 \\ \log. 300 = 2.477121 \\ \hline \log. 465.4 = 2.667822 \end{array}$$

£465.4 = amount of £300 for 9 years, at 5 per cent., compound interest; then, 465.4 - 300 = £165.4 interest.

$$\begin{array}{r} \text{from } £165.4 \\ \text{take } £135.0 \\ \hline £30.4 \\ 20 \\ \hline s.8.0 \\ £30 \quad 8 \quad 0 \text{ Ans.} \end{array}$$

14.

$$\begin{array}{r} \cancel{30000000} = £15840034.65799 \\ \hline 20 \\ s.13.1598 \\ 12 \\ d.1.9176 \\ 4 \\ \hline f'3.6704 \end{array}$$

£15840034 13 1½ Ans.

Exer.

15. Amount of £1 at 3 per cent. in 7 years=

$$\begin{array}{r}
 1.229874 \\
 1000 \\
 \hline
 £1229.874 \\
 20 \\
 \hline
 s.17.48 \\
 12 \\
 \hline
 d.5.76 \\
 4 \\
 \hline
 f.3.04
 \end{array}$$

£1229 17 5 $\frac{3}{4}$ Ans.16. £669 2 3=669.125;
then $1.03^7 = 1.33825$

0	0	0	0	1.33825	{ 1.06 amt. of £1 for 1 year.
1	1	1	1	1	
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>3382500000</u>	
1	2	3	4	3382255776	
<u>2</u>	<u>3</u>	<u>4</u>	<u>500000000</u>		
1	3	6	63709296		
<u>3</u>	<u>6</u>	<u>10000000</u>	<u>563709296</u>		
1	4	618216			
<u>4</u>	<u>100000</u>	<u>10618216</u>			
1	3036				
<u>500</u>	<u>103036</u>				
6					
<u>506</u>					

from 1.06=amount of £1 for 1 year,
take 1.

.06=interest;

$\begin{array}{ccc} \text{prin.} & \text{prin.} & \text{inter.} \\ \text{then } £1 : 100 :: .06 : £6 = \text{the rate.} \end{array}$
 Ans.

17. log. 1612.6992=3.207553

log. 1200=3.079181

divide log. 1.03=0.12837)0.128372(10 yrs. Ans.
.128370

HARMONICAL SERIES.

(Arithmetic, page 235.)

Exer.

1.
$$\frac{7 \times 9 \times 2}{7+9} = \frac{63}{8} = 7\frac{7}{8} \quad \text{Ans.}$$

2. If 3 means be required, as x, y, z ; y is a harmonic mean between a and b , and can be found by the rule; then x is a harmonic mean between a and y ; and z a harmonic mean between y and b .

Or, $x = \frac{4ab}{a+3b}$; $y = \frac{2ab}{a+b}$ and $z = \frac{4ab}{3a+b}$. In the present case, $y = \frac{2 \times 7 \times 63}{7+63} = \frac{882}{70} = 12\frac{3}{5} = 2\text{nd mean.}$
 $x = \frac{2 \times 7 \times 12\frac{3}{5}}{7+12\frac{3}{5}} = \frac{176\frac{4}{5}}{19\frac{3}{5}} = \frac{176.4}{19.6} = 9 = 1\text{st mean.}$
 $z = \frac{2 \times 12\frac{3}{5} \times 63}{12\frac{3}{5}+63} = \frac{1587\frac{6}{5}}{75\frac{3}{5}} = \frac{1587.6}{75.6} = 21 = 3\text{rd harmonic mean.}$

Or, 7 and 63 being changed into their reciprocals, $\frac{1}{7}$ and $\frac{1}{63}$ become the extremes of an equi-different series, of which the number of terms is 5; whence the common difference and intermediate terms can be found, the reciprocals of which will be the harmonic means required.

3. $\frac{1}{7}, \frac{1}{15}$ &c., form an equi-different series—

$$\frac{1}{7} - \frac{1}{15} = \frac{1}{15} - \frac{1}{21} = \frac{1}{21} - \frac{1}{28} = \frac{1}{28} - \frac{1}{35} = \text{common difference.}$$

Then $\frac{1}{7}, \frac{1}{15}, \frac{1}{21}, \frac{1}{28}, \frac{1}{35}$ form the series, the reciprocals of which are the harmonic series required, viz.—

$$\frac{1}{7} = 21, \frac{1}{15} = 15, \frac{1}{21} = 11\frac{2}{3}, \frac{1}{28} = 9\frac{1}{4}, \frac{1}{35} = 8\frac{1}{5}, \frac{1}{42} = 7. \quad \text{Ans.}$$

4. If the vibrations of $Do=256$, the vibrations of its descending octave=128, and the vibrations of sol in the descending scale are an equi-different mean (the reciprocals of an harmonic mean), between these $= \frac{128+256}{2} = \frac{384}{2} = 192 \quad \text{Ans.}$

Exer.

5.

$$Do=1, \text{ octave } Do=\frac{1}{2}$$

$$Sol=\frac{(1 \times \frac{1}{2}) \times 2}{1\frac{1}{2}}=\frac{2}{3}$$

$$Mi=\frac{(1 \times \frac{1}{2}) \times 2}{1\frac{1}{2}}=\frac{\frac{4}{3}}{\frac{3}{2}}=\frac{8}{9}$$

To find *Re* in second octave, if *Sol* be considered tonic, *Sol* its octave= $\frac{2}{3}$, and *Re* the 5th is a harmonic mean between them.

$$Re=\frac{(\frac{2}{3} \times \frac{1}{2}) \times 2}{\frac{2}{3} + \frac{1}{2}}=\frac{\frac{2}{3}}{1}=\frac{2}{3} \div 1=\frac{2}{3}$$

The double of $\frac{2}{3}=\frac{4}{3}=Re$ in the first scale.

The tonic being to its descending 5th in the ratio $\frac{3}{2} : 1$; *Fa*, the descending 5th of *Do*, is found thus—

$\frac{3}{2} : 1 :: 1 : \frac{2}{3}=Fa$ in a lower scale than those under consideration; then $\frac{3}{2} \div 2=\frac{3}{4}=Fa$, its octave, and $\frac{3}{4} \div 2=\frac{3}{8}=Fa$, the higher octave. The 4th from $Mi=\frac{8}{9} \times \frac{2}{3}=La$, in 1st octave, and $\frac{8}{9} \div 2=\frac{4}{9}=La$, in 2nd octave. The 5th from the tonic being $\frac{3}{2}$ thereof, the 5th from $Mi=\frac{8}{9} \times \frac{3}{2}=\frac{4}{3}=Si$ in the 1st octave; $\frac{4}{3} \div 2=\frac{2}{3}=Si$ in 2nd octave; then

First Octave.

Do	Re	Mi	Fa	Sol	La	Si
1	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{4}{5}$

Second Octaves the $\frac{1}{2}$'s of these:

$\frac{1}{2}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{8}$	$\frac{1}{3}$	$\frac{3}{10}$	$\frac{2}{5}$
---------------	---------------	---------------	---------------	---------------	----------------	---------------

These terms represent the lengths of the strings producing the respective notes; their reciprocals represent the vibrations, which are required by the question.

6. The reciprocals of 3, 4, viz.— $\frac{1}{3}$, $\frac{1}{4}$, &c., form an equi-different series; $\frac{1}{3}-\frac{1}{4}=\frac{1}{12}$ =common difference; then $\frac{1}{12}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$ =the series, the 4th and 5th terms of which are $\frac{1}{12}$ and $\frac{1}{8}$; the reciprocals of which are $12=12$ and $8=\text{infinity}$.

[Ans.]

ANNUITIES.

(Arithmetic, page 236.)

Exer.

1.

$$a=1, r=1.05, n=15$$

$$\log. 1.05 = .021189$$

15=index

$$\log. 1.05^{15} = 2.078909 = \overline{.317835}$$

from 2.078909

take 1.000000=first term

$$1.05-1 = .05 \overline{)1.078909}$$

21.57818=sum

80

1726.2544=amount

20

s. 5.088

12

d. 1.056

£1726 5 1 Ans.

2.

$$a=1, r=1.04, n=7$$

$$\log. 1.04 = .017033$$

7 index

$$\log. 1.04^7 = 1.315924 = \overline{.119231}$$

from 1.315924

take 1.000000=1st term

$$1.04-1 = .04 \overline{)1.315924}$$

7.8981

50

394.905=amount

20

s. 18.100

12

d. 1.200

£394 18 1½ Ans.

Exer.

3.

$$\log. 1.035 = .01494$$

8=index

$$\log. 1.035^8 = 1.3168 = .11952$$

from 1.3168

take 1.0000

$$1.035 - 1 = .035 \overline{) 3168}$$

$$9.0514 = \text{sum}$$

$$119.11.3$$

$$\overline{814626}$$

$$90514.$$

$$90514..$$

$$10s. \text{ is } £\frac{1}{2}, \dots 45257$$

$$1s. 3d. \text{ is } \frac{1}{8} \dots 5657$$

$$£1082.208 = \text{amount}$$

$$\overline{20}$$

$$s.4.160$$

$$12$$

$$£1082 \quad 4 \quad 1.92 \text{ Ans.} \quad \overline{d.1.92}$$

4.

$$\log. 1.06 = .025306$$

9=index

$$\log. 1.06^9 = 1.68948 = .227754$$

from 1.68948

take 1.00000

$$1.06 - 1 = .06 \overline{) .68948}$$

$$11.4913$$

$$28.8.9$$

$$\overline{919304}$$

$$229826$$

$$5s. \text{ is } £\frac{1}{4} \dots 28728$$

$$2s. 6d. \text{ is } \frac{1}{2} \dots 14364$$

$$1s. 3d. \text{ is } \frac{1}{4} \dots 7182$$

$$£326.7838 = \text{amount}$$

$$\overline{20}$$

$$s.15.676$$

$$12$$

$$15 \quad 8.112 \text{ Ans.} \quad \overline{d.8.112}$$

Exer.

5. $\log. 1.06 = .025306$
 $\log. 1.06^{14} = 2.260916 = .354284$ $14 = \text{index}$
 $1.06 - 1 =$
 $\begin{array}{r} \text{from } 2.260916 \\ \text{take } 1.000000 \\ \hline .06 \overline{) 1.260916} \\ 21.01526 = \text{sum} \\ 101.2 \cdot 2 \\ \hline 2101526 \\ 2101526 \\ \hline \end{array}$
 $2s. \text{ is } £ \frac{1}{10} \dots 2101526$
 $2d. \text{ is } \frac{1}{10} \dots 175127$
 $£2124.817913 =$
 $£2123 \ 16 \ 4\frac{1}{4} \text{ Ans.}$

6. $\log. 1.045 = .019116$
 $\log. 1.045^{25} = 3.00538 = .4779$
 $1.045 - 1 =$
 $\begin{array}{r} \text{from } 3.00538 \\ \text{take } 1.00000 \\ \hline .045 \overline{) 2.00538} \\ £44.564 = \text{sum} \\ 80 \\ \hline £3565.12 \\ 20 \\ \hline s. 2.4 \\ 12 \\ \hline d. 4.8 \end{array}$
 $£3565 \ 2 \ 4.8 \text{ Ans.}$

Exer.

7.

$$\log. 1.06 = .025306$$

10

$$\log. 1.06^{10} = 1.790855 = .25306$$

from 1.790855

take 1.000000

$$1.06 - 1 = .06 \quad \cdot 790855$$

13.1809

200

$$\pounds 2636.18 = \text{amount of}$$

£200 in 10 years, at 6 per cent. per annum.

$1.02956301 = \text{amount of } \pounds 1 \text{ in } \frac{1}{2} \text{ a year.}$

$\sqrt[2]{1 \times 1.02956301} = \text{amount } \pounds 1 \text{ in } \frac{1}{4} \text{ year.}$

$\sqrt[2]{1 \times 1.02956301} = \sqrt[2]{1.02956301} = 1.014673 =$
amount of £1 in $\frac{1}{4}$ year.

$$\log. 1.014673 = .006326$$

40

$$\log. 1.014673^{40} = 1.790772 = .25304$$

from 1.790772

take 1.000000

$$1.014673 - 1 = .014673 \quad \cdot 790772$$

53.893 = sum.

50

$$\pounds 2694.65 = \text{amount of}$$

£50, quarterly, in 10 years, at 6 per cent.

from £2694.65

take £2636.18

£58.47

20

s.9.4

12

d.4.8

£58 9s. 4.8d. Answer in favour of quarterly payments.

Exer.
8.

$$\log. 1.04 = .017033$$

$$\begin{array}{r} 3 \\ 2) .051099 \end{array}$$

$$\log. 1.04^{\frac{1}{2}} = 1.06059 = .0255495$$

$$\log. 1.4233 = 1.53297$$

$$\text{from } 1.4233$$

$$\text{take } 1.0000$$

$$1.06059 - 1 = .06059) .4233$$

$$6.9863 = \text{sum of series,}$$

whose first term=1, ratio=1.06059, and number
of terms=6, (for 6×18 months=9 years).

$$6.9863$$

$$240$$

$$2794520$$

$$139726$$

$$\pounds 1676.712$$

$$20$$

$$s. 14.24$$

$$12$$

$$d. 2.88$$

$$4$$

$$f. 3.52 \quad [\pounds 1676 \ 14 \ 2\frac{3}{4} \text{ Ans.}]$$

10. 35.719251 = amount of $\pounds 1$ annuity at the given rate and time (by table). 2.785963 = amount of $\pounds 1$ at compound interest at the given rate and time (per table). $35.719251 \div 2.785963 = \pounds 12.82115$ = present value of annuity of $\pounds 1$ at the given rate and time; then

$$12.82115$$

$$14$$

$$\pounds 179.49610$$

$$20$$

$$s. 9.922$$

$$12$$

$$d. 11.064 \quad [\pounds 179 \ 9 \ 11 \text{ Ans.}]$$

Exer.

9. Amount of an annuity of £1 for 7 years, at 6 per cent.=8.393837. Amount of £1, at compound interest for 7 years, at 6 per cent.=1.50363. Then $\frac{8.393837}{1.50363} = 5.582382$ = present value of £1 due 7 years hence, at 6 per cent. per annum.

$$\begin{array}{r}
 5.582382 \\
 43 \cdot 1 \cdot 7 \\
 \hline
 16747146 \\
 22329528 \cdot \\
 1s. \text{ is } £\frac{1}{20} \dots 279119 \\
 6d. \text{ is } \frac{1}{2} \dots 139559 \\
 1d. \text{ is } \frac{1}{6} \dots 23259 \\
 \hline
 £240 \cdot 484363 \\
 20 \\
 \hline
 s.9 \cdot 68726 \\
 12 \\
 \hline
 d.8 \cdot 24712 \\
 £240 \quad 9 \quad 8 \text{ Ans.}
 \end{array}$$

Exer.

11. $\frac{5.52553}{1.47611} = 4.329474$ = present value of £1 annuity for 5 years, at 5 per cent. per annum; then 4.329474

$$\begin{array}{r}
 435 \\
 \hline
 21647370 \\
 17317896 \\
 8658948 \\
 \hline
 £1060 \cdot 72113 \\
 20 \\
 \hline
 s.14 \cdot 4226 \\
 12 \\
 \hline
 d.5 \cdot 0712 \\
 £1060 \quad 14 \quad 5 \text{ Ans.}
 \end{array}$$

Exer.

12. $\frac{8.286127}{1.480147} = 8.110897$ = present value of £1 for 10 years at 4 per cent. per annum.

$$\begin{array}{r}
 8.110897 \\
 455 \\
 \hline
 40554485 \\
 40554485 \\
 32443588 \\
 \hline
 £3690 \cdot 458135 \\
 20 \\
 \hline
 s.9 \cdot 1627 \\
 12 \\
 \hline
 d.1 \cdot 9524 \\
 4 \\
 \hline
 f.3 \cdot 8096 \\
 £3690 \quad 9 \quad 1\frac{1}{4} \text{ Ans.}
 \end{array}$$

Exer.

$$\begin{array}{r}
 \text{£} \quad \text{£} \quad \text{£} \quad \text{s.} \\
 13. \quad 3 : 100 :: 700 \quad 10 \\
 \quad \quad \quad 100 \\
 \quad \quad \quad \underline{3)70050} \\
 \text{Ans. } \text{£}23350
 \end{array}$$

$$\begin{array}{r}
 \text{£} \quad \text{£} \quad \text{£} \quad \text{s.} \\
 14. \quad 4 : 100 :: 280 \quad 15 \\
 \quad \quad \quad 100 \\
 \quad \quad \quad \underline{4)28075} \\
 \text{Ans. } \text{£}7018.15s.
 \end{array}$$

$$\begin{array}{r}
 5. \quad 2.18447 = 10.837767 = \\
 \text{present worth of } \text{£}1; \\
 \text{then } 10.837767 \\
 \quad \quad \quad 120 \\
 \underline{\text{£}1300.53204} \\
 \quad \quad \quad 20 \\
 \text{s. } 10.6408 \\
 \quad \quad \quad 12 \\
 \underline{\text{d. } 7.6896} \\
 \quad \quad \quad 4 \\
 \underline{\text{f. } 2.7584} \\
 \text{£}1300 \quad 10 \quad 7\frac{1}{2} \text{ Ans.}
 \end{array}$$

Exer.

$$\begin{array}{r}
 \text{£} \quad \text{s.} \quad \text{s.} \quad \text{d.} \quad \text{£} \quad \text{s.} \quad \text{d.} \\
 16. \quad 1 \quad 15-12 \quad 6=1 \quad 2 \quad 6 \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 10 \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \underline{11 \quad 5 \quad 0} \\
 \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad 7
 \end{array}$$

£78 15 = the amount to be fined down for 39 years.

$$\log. 1.04 = .017033$$

$$\log. 4.616223 = .664287$$

$$\begin{array}{r}
 \text{from } 4.616223 \\
 \text{take } 1.000000
 \end{array}$$

$$\begin{array}{r}
 1.04-1=.04) \quad 3.616223 \\
 \underline{90.405575}
 \end{array}$$

= the amount of £1 annuity for 39 years.
 $\frac{19.584317}{2.18447} = 19.584317 =$
 present value of £1.

$$\begin{array}{r}
 19.584317 \\
 \quad \quad 78.75 \\
 \underline{97921585} \\
 137090219 \\
 156674536 \\
 137090219 \\
 \underline{\text{£}1542.26496375} \\
 \quad \quad \quad 20 \\
 \text{s. } 5.299275 \\
 \quad \quad \quad 12 \\
 \underline{\text{d. } 3.5913} \\
 \quad \quad \quad 4 \\
 \underline{\text{f. } 2.3652}
 \end{array}$$

$$\text{£}1542 \quad 5 \quad 3\frac{1}{2} \text{ Ans.}$$

Exer.

17. 2.956301 = rate per half year, at 6 per cent. per annum (see *Ex.* 2, page 236); then

$$2.956301 : 100 :: 100$$

$$\frac{100}{2.956301} = \underline{\underline{\pounds 3382.60583}}$$

$$\begin{array}{r} 20 \\ \hline s. 12.1766 \end{array}$$

$$\begin{array}{r} 12 \\ \hline d. 1.3992 \end{array}$$

$$\begin{array}{r} 4 \\ \hline f. 1.5968 \end{array}$$

$$\pounds 3382 \ 12 \ 1\frac{1}{4} \text{ Ans.}$$

18. $\sqrt[2]{1.04} = 1.0198039$ = ratio per half year,

from 1.0198039

take 1

$$\frac{.0198039}{1.0198039};$$

then $\frac{1.0198039}{.0198039} = 50.49510449$ = present value of an annuity of $\pounds 1$, payable half yearly,

$$\frac{50.49510449}{10}$$

$$5.049510449 = \text{present value of}$$

$\pounds 20$, payable half yearly; then $\frac{50.49510449}{5.049510449} =$

25.247552245 = number of years purchase = present value divided by the annuity. Ans.

19. From 12.82115 = present value of $\pounds 1$ for 21 years,

take 5.786372 = do. do. for 7 years.

$$\frac{7.034778}{500}$$

$$\frac{500}{7.034778}$$

$$\pounds 3517.398 = \text{reversionary value.}$$

$$\begin{array}{r} 20 \\ \hline s. 7.78 \end{array}$$

$$\begin{array}{r} 12 \\ \hline d. 9.36 \end{array}$$

$$\begin{array}{r} 4 \\ \hline f. 1.44 \end{array}$$

$$\pounds 3517 \ 7 \ 9\frac{1}{4} \text{ Ans.}$$

Exer.

20. From 14.451114 = present value of £1 for 22 yrs.
take 8.110895 = do. do. for 10 yrs.

6.340219

15 · 12 · 6

95103285

10s. is £ $\frac{1}{2}$ 31701095

2s. 6d. is $\frac{1}{4}$ 7925273

£99.0659218

20

s.1.318436

12

d.3.821232

4

f.3.284928

[£99 1 3 $\frac{3}{4}$ Ans.

21. From 12.783355 = present value of £1 for 25 yrs.
take 2.673011 = do. do. for 3 yrs.

10.110344

170 · 11 · 4

1718.75848

10s. is £ $\frac{1}{2}$ 5.055172

1s. 4d. is $\frac{1}{5}$.6740229

£1724.4876749

20

s.9.753498

12

d.9.041976

[£1724 9 9 Ans.

23. 4 : 100 :: 100 £2500 = present value of £100 in
perpetuity ; then
from 2500

take 743.5331 = present value of £100 for 9 yrs.

£1756.4669 = reversionary value.

20

s.9.338

12

d.4 056

[£1756 9 4 Ans.

Exer.

22. From 4.579708 = present value of £1 for 5 years,
 take 1.913470 = do. do. for 2 years.
 $2.666238 \times 200 = £533.2476 = £533 \ 4 \ 11\frac{1}{4}$
 = reversionary value. Ans.

24. $\log. 1.05 = .021189$
 $\underline{50}$
 $\log. 1.05^{50} = 11.467 = \underline{1.05945}$
 from 11.467 = amt. of £1 in 50 years,
 take 1 at compound interest.
 $1.05 - 1 = .05 \overline{) 10.467}$
 209.34 = amt. of £1 an. for 50 yrs.
 $19.434 = 18.255864$ = present value of annuity.
 $.85 = 20$ = present value of perpetuity.
 from 20
 take 18.255864
 $\underline{1.744136}$
 40
 $£69.76544$ = purchase, nearly.
 $69.76544 = 1.744136$ years' purchase. Ans.

LIFE ANNUITIES.

(Arithmetic, page 241.)

1. 10.11 = value of £1 annuity on a life aged 45 yrs.
 36 = annuity.
 $£363.96 = £363 \ 19 \ 2\frac{1}{4}$ Ans.
2. 4.962 = value of £1 on a life aged 75 years.
 150 = annuity.
 $£744.3 = £744 \ 6 \ 0$ Ans.
3. $10s.$ is $£\frac{1}{2} \dots \frac{56}{28.0}$
 $2s. \ 6d.$ is $\frac{1}{4} \dots 7.$
 $£35$ = profit rent;
 then 11.837 = value of £1 on a life aged 40 years.
 $\underline{35}$
 $£414.295 = £414 \ 5 \ 10\frac{3}{4}$ Ans.

Exer.

4. From 9.385073 = present value of £1 for 12 years,
take 6.361 = present value of £1 on a life aged 70 yrs.

$$\frac{3.024073}{500.5} \times 48 = £145.1555 = £145 \ 3 \ 1\frac{1}{4}$$

purchase money. Ans.

5. 12.063 = present value of £1 on a life aged 25 yrs.
 500.5

$$\frac{£6037.5315}{£6 : £100 :: £500 \ 10s. : £8341.66666} = \text{present value of } £500.10 \text{ in perpetuity; then}$$

$$\text{from } 8341.66666$$

$$\text{take } 6037.5315$$

$$£2304.13516 = £2304 \ 2 \ 8\frac{1}{4} \text{ Ans.}$$

6. $\frac{1}{25} = 20$ = present value of £1 in perpetuity.

from 20

$$\text{take } 9.193 = \text{value of } £1 \text{ an. on a life aged 56 yrs.}$$

$$\frac{10.807}{20+1} = .514619 \times 150 = 77.19285, \text{ and } [\text{Ans.}]$$

$$77.19285 = £8.3969 = £8 \ 7 \ 11\frac{1}{4} = \text{annual payments.}$$

7. $\frac{1}{16} = 16.66666$ = value of £1 in perpetuity; then
from 16.66666

$$\text{take } 5.716 = \text{value of annuity to a life 70 years.}$$

$$\frac{10.95066}{1+16.66666} = .6198489, \times 1000 = £619.8489 =$$

$$£619 \ 16 \ 11\frac{3}{4} = \text{Amount. Ans.}$$

8. 11.118487 = present value of £1 annuity for 15 yrs.
20

$$222.36974 = \text{present value of } £20 \text{ an. for 15 years.}$$

from 25 = present value of £1 in perpetuity, at 4 per cent.

$$\text{take } 12.283 = \text{present value of } £1 \text{ during life, age 45 years.}$$

$$\frac{12.717}{25+1} = .489115;$$

$$\text{then } .489115 \times 222.36974 = 108.76437528, \text{ and}$$

$$£108.76437528 = £8.85487 = £8 \ 17 \ 1 \text{ annual payment. Ans.}$$

PERMUTATIONS.

(Arithmetic, page 245.)

Exer.

1. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40320$ Ans.
2. $1 \times 2 \times 3 \times 4 \times 5 = 120$ Ans.
3. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 362880$ Ans.
4. 1×2 , &c., to $10 = 3628800 \times 11 \times 12 = 479001600$
days; then

$$479001600 = 1312333 \overset{\text{years.}}{+} 55 \overset{\text{days.}}{\text{Ans.}}$$

5. $22 \times 23 \times 24 \times 25 \times 26 = 7893600$ Ans.
6. $50 \times 49 \times 48 = 117600$ Ans.
7. $52 \times 51 \times 50 \times 49 \times 48 = 311875200$ Ans.
8. $9 \times 8 \times 7 \times 6 = 3024$ Ans.

SIMPLE SUPPOSITION.

(Arithmetic, page 246.)

Exer.

1. Suppose 40

$$\frac{1}{4} \dots 10$$

$$\frac{1}{5} \dots 8$$

$$\overline{58} : 87 :: 40$$

$$40$$

$$58 \overline{) 3480}$$

$$\overline{60} \text{ Ans.}$$

Exer.

2. Suppose 30

$$\frac{4}{5} \times 30 = 24$$

$$6$$

$$\overline{144}$$

$$5 \times 30 = 150$$

$$\text{dif.} = 6 \text{ should be} = 5$$

$$6 : 5 :: 30 : 25 \text{ years,}$$

the Ans.

3.

Suppose 180,

$$\frac{1}{4} = 45$$

$$\frac{2}{3} = 120$$

$$\frac{1}{20} = 9$$

$$\overline{174}$$

$$\text{then } 180 - 174 = 6, \text{ and}$$

$$6 : 8 :: 180 : 240 \text{ Ans.}$$

Exer.

4. Suppose 20 = first part,

$\frac{1}{2} = 10 = \frac{1}{3}$ of 2nd, $\frac{1}{4}$ of 3rd, and $\frac{1}{5}$ of 4th part.

$$3 \times 10 = 30 = 2\text{nd},$$

$$4 \times 10 = 40 = 3\text{rd},$$

$$5 \times 10 = 50 = 4\text{th};$$

$$\text{then } 20 + 30 + 40 + 50 = 140,$$

$$1\frac{1}{4}^{\circ} : 1\frac{1}{3}^{\circ} :: 20 : 30; \text{ then}$$

$\frac{2}{3} = 15 = \frac{1}{3}$ first, $\frac{1}{3}$ second, $\frac{1}{4}$ third, and $\frac{1}{5}$ fourth;

$$\text{then } 15 \times 3 = 45 = 2\text{nd},$$

$$15 \times 4 = 60 = 3\text{rd},$$

$$15 \times 5 = 75 = 4\text{th}.$$

$$\text{Proof—} 30 + 45 + 60 + 75 = 210.$$

DOUBLE SUPPOSITION.

(Arithmetic, page 250.)

1. Suppose 21 and 9;

$$\text{then } 9 \div 6 = 1\frac{1}{2}$$

$$21 \div 3 = 7$$

$$\overline{8\frac{1}{2}}$$

should be 8

$$\text{error } +\frac{1}{2}$$

Suppose 24 and 6;

$$\text{then } 6 \div 6 = 1$$

$$24 \div 3 = 8$$

$$\overline{9}$$

should be 8

$$\text{error } +1$$

then $\frac{1}{2}$ (dif. of errors) : 3 (24-21) :: 1 (9-8) : 6;

then. the errors being in excess,

$$24 - 6 = 18 \text{ the greater,}$$

$$30 - 18 = 12 \text{ the less. Ans.}$$

2. Suppose 10; then $10 \times 5 = 50$

should be 38

$$\text{error } +12$$

Suppose 6; then $6 \times 5 = 30$

should be 34

$$\text{error } -4$$

then $16 (= 12 + 4) : 4 (= 10 - 6) :: 12 : 3$ correction;

$$\text{then } 10 - 3 = 7 \text{ Ans.}$$

*Exer.*3. $6 \times 14 = 84$ coats.

$$\begin{array}{r}
 \text{Suppose } 60 \text{ at } 2s. \text{ } 6d. = \text{£}7 \ 10 \ 0 \\
 \text{and } 24 \text{ at } 1s. \text{ fine} = \quad 1 \ 4 \ 0 \\
 \hline
 \qquad \qquad \qquad 6 \ 6 \ 0 \\
 \text{should be } 7 \ 0 \ 0 \\
 \text{error } -\text{£}0 \ 14 \ 0
 \end{array}$$

$$\begin{array}{r}
 \text{Suppose } 72 \text{ at } 2s. \text{ } 6d. = \text{£}9 \ 0 \ 0 \\
 \text{and } 12 \text{ at } 1s. \text{ fine} = \quad 0 \ 12 \ 0 \\
 \hline
 \qquad \qquad \qquad 8 \ 8 \ 0 \\
 \text{should be } 7 \ 0 \ 0 \\
 \text{error } +\text{£}1 \ 8 \ 0
 \end{array}$$

$$\begin{array}{r}
 \text{then } 42 : 12 :: 14 \\
 \qquad 3 \quad 4 \quad 1
 \end{array}$$

Then, as 14 is an error of defect, $60 + 4 = 64 =$
 number of coats which fit, and $84 - 64 = 20 =$
 number of misfits. Ans.

4. Suppose 2 = 1st, then $\frac{3}{4}$ = common difference,
 and 8 = number of terms;

$$\begin{array}{r}
 \text{sum } \frac{8}{2}(2 \times 2 + (7 \times \frac{3}{4})) = 4 \times 9\frac{1}{4} = 37 \\
 \text{should be } 31 \\
 \text{error } +6
 \end{array}$$

Suppose 1 = 1st term, $\frac{3}{4}$ common difference;

$$\begin{array}{r}
 \text{sum } \frac{8}{2}(2 \times 1 + 7 \times \frac{3}{4}) = 4 \times 7\frac{1}{4} = 29 \\
 \text{should be } 31 \\
 \text{error } -2
 \end{array}$$

$8 : 1 :: 6 : \frac{3}{4}$ = excess of 1st supposition; then
 $2 - \frac{3}{4} = 1\frac{1}{4}$ = 1st term, and $1\frac{1}{4} + 2 + 2\frac{3}{4} \times 3\frac{1}{2} + 4\frac{1}{4} + 5 +$
 $5\frac{3}{4} + 6\frac{1}{2} = 31$ Ans.

Exer.

5. Suppose 15 and 5 ;

then $5^2=25$ error $-\overline{10}$

Suppose 17 and 3 ;

then $3^2=9$ error $+\overline{8}$

Then 18, (sum of errors) : 2 :: 10 : $1\frac{1}{3}$, and
 $15+1\frac{1}{3}=16\frac{1}{3}$.

Again—

Suppose $16\frac{1}{3}$ and $3\frac{2}{3}$;then $3\frac{2}{3}^2=15\frac{10}{9}$ error $+\frac{10}{9}$ Suppose $15\frac{2}{3}$ and $4\frac{1}{3}$;then $4\frac{1}{3}^2=16\frac{2}{3}$ error $-\overline{1\frac{1}{3}}$

$2(=\frac{10}{9}+1\frac{1}{3}) : \frac{2}{3} :: \frac{10}{9} : \frac{10}{9} ;$ then $16\frac{1}{3}-\frac{10}{9}=16\frac{1}{3}$
 greater approximation, and $3\frac{2}{3}$ less approxi-
 mation. By further repeating the process we
 will, at length, find 16 and 4 the true numbers.

6. Suppose 2=root ;

then $2^5=32$

should be 100

error $-\overline{68}$

Suppose 3=root ;

then $3^5=243$

should be 100

error $+\overline{143}$

211 (sum of error), : 1 :: 68 : .32 to be added to
 2(=first supposed number).

Again—

Suppose 2.32=root ;

 $2.32^5=67.210933$

should be 100.

error $-\overline{32.789067}$

Suppose 3=root ;

 $3^5=243$

should be 100

error $+\overline{143}$

175.789 (sum of errors), : .68 :: 32.789 : .12 to
 be added to 2.32.

Again—

Suppose 2.44 ;

then $2.44^5=86.4866$

should be 100

error $-\overline{13.5134}$

Suppose 3—this

as before,

gives error

 $+\overline{143}$

156.5134(=sum of errors) : .56 :: 13.5134 : .04
 to be added to 2.44, gives 2.48.

*Err.*6—*continued.*

Again—

Suppose $2.48 = 100^{\frac{1}{2}}$

$$2.48^{\circ} = 93.812$$

$$\text{error } -6.188$$

$$+143.$$

And 3, as before
gives +143

$$\frac{149.188}{149.188} : .52 :: 6.188 : .02156 \text{ to}$$

be added to 2.48, gives 2.50156.

Again—

Suppose $2.5 = 100^{\frac{1}{2}}$

$$2.5^{\circ} = 97.656$$

$$\text{error } -2.344$$

Suppose $2.48 = 100^{\frac{1}{2}}$

$$2.48^{\circ} = 93.812$$

$$\text{error } -6.188$$

3.844 (difference of errors) : .5 :: 2.344 : .0304
to be added to 2.5, gives 2.5304.

Again—

Suppose $2.53 = 100^{\frac{1}{2}}$

$$3.53^{\circ} = 103.2579$$

$$\text{error on } 2.53 = +3.2579$$

$$\text{error, as above, on } 2.5 = -2.344$$

$$\text{dif. of numbers} = .03, \quad 5.6019 = \text{sum, or errors.}$$

5.6019 : .03 :: 3.2579 : .01744 to be subtracted
from 2.53 gives 2.51256 Ans.

7.

Suppose 321 ; sum of digits 6.

$$6 \times 48 = 288$$

$$-33$$

Suppose 876 ; sum of digits 21.

$$21 \times 48 = 1008$$

$$+132$$

165 (sum of errors), : 555 (difference of numbers),
:: 33 : 111 ; then $111 + 321 = 432$. Ans.

Exer.

8.

$$\left. \begin{array}{l} 1+1+1=11 \\ 1+1+1=17 \\ 1+1+1=27 \end{array} \right\} \text{ and } 11+17+27=73\frac{1}{2}.$$

In order to avoid fractions we will suppose numbers that are multiples of these denominators—

Suppose 24, 60, & 120, then

$$\frac{13 \times 24}{12} + \frac{47 \times 60}{60} + \frac{37 \times 120}{60} =$$

$$26 + 47 + 74 = 147$$

$$\text{should be} = 73\frac{1}{2}$$

$$\text{error} + 73\frac{1}{2}, \text{ or } 73\cdot5$$

Suppose 36, 90, & 180; then

$$\frac{13 \times 36}{12} + \frac{47 \times 90}{60} + \frac{37 \times 180}{60} =$$

$$39 + 70\cdot5 + 111 = 220\cdot5$$

$$\text{should be} = 73\cdot5$$

$$\text{error} + 147$$

147	147	147	73·5	73·5	73·5
24	60	120	36	90	180
3528	8820	17640	2646	6615	13230

$$147 - 73\cdot5 = 73\cdot5$$

$$3528 - 2646 = 882$$

$$8820 - 6615 = 2205$$

$$17640 - 13230 = 4410$$

$$\left. \begin{array}{l} \text{then } 882 \div 73\cdot5 = 12 \\ 2205 \div 73\cdot5 = 30 \\ 4410 \div 73\cdot5 = 60 \end{array} \right\} =$$

the number required. Ans.

Exer.

9. Since their sum is to the greater as 8 : 5, therefore the greater is to the less as 5 : 3; and the square of the greater is to the square of the less as 25 : 9. Let 25 and 9 be the squares of the numbers— then $25 - 9 = 16$
should be = 144

error -128 Suppose 100, and 36; then $100 - 36 = 64$

should be = 144

error -80

80	80	128	128
25	9	100	36
<u>2000</u>	<u>720</u>	<u>12800</u>	<u>4608</u>

$$128 - 80 = 48$$

$$12800 - 2000 = 10800$$

$$4608 - 720 = 3888$$

then $10800 \div 48 = 225$ } = the squares of the
 $3888 \div 48 = 81$ } numbers;
 and $\sqrt{225} = 15$ } = the numbers. Ans.
 $\sqrt{81} = 9$ }

10. Suppose 20 half-crowns and 16 half-sovereigns—
then $30 \times 20 = 600d.$

$$120 \times 16 = 1920$$

$$\underline{2520}$$

should be $2610 = \text{£}10 \ 17 \ 6.$ error -90

Next suppose 12 half-crowns and 24 half-sovs.—

$$\text{then } 30 \times 12 = 360d.$$

$$120 \times 24 = 2880$$

$$\underline{3240}$$

should be 2610

error $+630$

$$\begin{array}{l} 630 \times 20 = 12600 \\ 630 \times 16 = 10080 \\ 90 \times 12 = 1080 \\ 90 \times 24 = 2160 \end{array} \left. \vphantom{\begin{array}{l} 630 \times 20 \\ 630 \times 16 \\ 90 \times 12 \\ 90 \times 24 \end{array}} \right\} \text{then } \left\{ \begin{array}{l} 12600 + 1080 = 13680 \\ 10080 + 2160 = 12240 \end{array} \right.$$

$$13680 \div 720 (= 630 + 90) = 19 \text{ half-crowns.}$$

$$12240 \div 720 = 17 \text{ half-sovereigns. Ans.}$$

*Exer.*11. Let $x=4$;

then $x^2=16$

$10x^2=160$

$5x=20$

$\overline{244}$

should be $\overline{260}$

error $\overline{-16}$

$16 \times 4.5 = 72$

$56.125 \times 4 = 224.5$

$\overline{296.5 \div 72.125 (=16+56.125) = 4.11}$

first approximation.

Suppose 4.11 and 4.12;

$4.11^2 = 69.426531$

$10 \times 4.11^2 = 168.921$

$5 \times 4.11 = 20.55$

$\overline{258.897531}$

should be $\overline{260}$

error $\overline{-1.102469}$

$1.102469 \times 4.12 = 4.54217228$

$\cdot 278528 \times 4.11 = 1.14475008$

$\overline{5.68692236}$

$5.68692236 \div 1.380997 (=1.102469 + .278528) =$

$4.117983 = \text{second approximation. Ans. nearly.}$

By repeating the process, using 4.117 and 4.12 would probably give the answer in the book.

12. Product + the sum = 47, sum of second powers—
sum of the numbers = 62; then $62 + 47 = 109$.

Let 9 and 11 be the Nos.

then $9^2 = 81$

$11^2 = 121$

$9 \times 11 = 99$

$\overline{301}$

should be $\overline{109}$

error $\overline{+192}$

Suppose 6 and 8.

then $6^2 = 36$

$8^2 = 64$

$6 \times 8 = 48$

$\overline{148}$

should be $\overline{109}$

error $\overline{+39}$

Exer.

12—continued.

$ \begin{array}{r} 192 \times 6 = 1152 \\ 39 \times 9 = 351 \\ 192 - 39 = 153 \overline{)801} (5 \cdot 2 \\ \underline{765} \\ 360 \\ \underline{306} \end{array} $	$ \begin{array}{r} 192 \times 8 = 1536 \\ 39 \times 11 = 429 \\ 153 \overline{)1107} (7 \cdot 2 \\ \underline{1071} \\ 360 \\ \underline{306} \end{array} $
--	--

Again, suppose 6 and 8, also 5·2 and 7·2—

Error on 6 & 8 = +39, $5 \cdot 2^2 = 27 \cdot 04$

$7 \cdot 2^2 = 51 \cdot 84$

$5 \cdot 2 \times 7 \cdot 2 = 37 \cdot 44$

$\underline{116 \cdot 32}$

should be 109·

$\underline{+7 \cdot 32}$

$39 \times 5 \cdot 2 = 202 \cdot 8$

$7 \cdot 32 \times 6 = 43 \cdot 92$

$\underline{158 \cdot 88}$

$39 \times 7 \cdot 2 = 280 \cdot 8$

$7 \cdot 32 \times 8 = 58 \cdot 56$

$\underline{222 \cdot 24}$

$158 \cdot 88 \div 31 \cdot 68 (= 39 - 7 \cdot 32) = 5 \cdot 01,$

$222 \cdot 24 \div 31 \cdot 68 = 7 \cdot 01. \text{ Again—}$

Suppose 5·2 and 7·2, also 5·01 and 7·01—

Error on 5·2 & 7·2 = +7·32, $5 \cdot 01^2 = 25 \cdot 1001$

$7 \cdot 01^2 = 49 \cdot 1401$

$5 \cdot 01 \times 7 \cdot 01 = 35 \cdot 1201$

$\underline{109 \cdot 3603}$

should be 109·

$\underline{\text{error } +3 \cdot 603}$

$7 \cdot 32 \times 5 \cdot 01 = 36 \cdot 6732$

$3 \cdot 603 \times 5 \cdot 2 = 1 \cdot 87356$

$\underline{34 \cdot 79964}$

$7 \cdot 32 \times 7 \cdot 01 = 51 \cdot 3132$

$3 \cdot 603 \times 7 \cdot 2 = 2 \cdot 59416$

$\underline{48 \cdot 71904}$

 $7 \cdot 32 - 3 \cdot 603 = 6 \cdot 9597. \text{ Then we have—}$

$$\left. \begin{array}{l}
 34 \cdot 79964 \div 6 \cdot 9597 = 5 \cdot 0001 \\
 48 \cdot 71904 \div 6 \cdot 9597 = 7 \cdot 0001
 \end{array} \right\} = 3\text{rd approximation.}$$

The decimal part of this third approximation may be considered as nothing; 5 and 7 are therefore the numbers required. Ans.

CONTINUED FRACTIONS.

(Arithmetic, page 254.)

Exer. 1.

$$\begin{array}{r}
 1\text{---}00000000)3\text{---}14159265(3 \\
 \underline{300000000} \\
 14159265)100000000(7 \\
 \underline{99114855} \\
 885145)14159265(15 \\
 \underline{13277175} \\
 882090)885145(1 \\
 \underline{882090} \\
 3055)882090(288 \\
 \underline{879840} \\
 2250
 \end{array}$$

Exer. 2.

Diameter = 1.0000000, solidity = 5235988. Then diameter, : solid, :: 1.0000000 : 5235988. Put the terms of second ratio into the form of a fraction and proceed according to the rule for finding a common measure; thus—

$$\begin{array}{r}
 \overline{5235988})10000000(1 \\
 \underline{5235988} \\
 4764012)5235988(1 \\
 \underline{4764012} \\
 471976)4764012(10 \\
 \underline{4719760} \\
 44252)471976(10 \\
 \underline{442520} \\
 29456)44252(1 \\
 \underline{29456} \\
 14796)29456(1 \\
 \underline{14796} \\
 14660)14796(1 \\
 \underline{14660} \\
 136 \\
 \&c.
 \end{array}$$

Exer. 3.

$$\begin{array}{r}
 \overline{19258} \\
 19258 \overline{)19500(1} \\
 \underline{19258} \\
 242 \overline{)19258(79} \\
 \underline{19118} \\
 140 \overline{)242(1} \\
 \underline{140} \\
 102 \overline{)140(1} \\
 \underline{102} \\
 38 \overline{)102(2} \\
 \underline{76} \\
 26 \overline{)38(1} \\
 \underline{26} \\
 12 \overline{)26(2} \\
 \underline{24}
 \end{array}$$

Exer. 4.

59160798

 $\overline{10000000} 59160798(5$

50000000

 $\overline{9160798} 10000000(1$

9160798

 $\overline{839202} 9160798(10$

8392020

therefore—

 $\sqrt{35} = 5 + \frac{1}{1} + \frac{1}{10} + \frac{1}{1} + \frac{1}{768778} 839202(1$

768778

 $\overline{70424} 768778(10$

704240

 $\overline{64538} 70424(1$

64538

 $\overline{5986} 64538(10$

59860

&c.

37416574

 $\overline{10000000} 37416574(3$

30000000

 $\overline{7416574} 10000000(1$

7416574

 $\overline{2583426} 7416574(2$

5166852

therefore—

 $\sqrt{14} = 3 + \frac{1}{1} + \frac{1}{2} + \frac{1}{8} + \frac{1}{1} + \frac{1}{2249722} 2583426(1$

2249722

 $\overline{333704} 2249722(6$

2002212

 $\overline{247510} 333704(1$

247510

 $\overline{86194} 247510(2$

172388

99498744

 $\overline{10000000} 99498744(9$

90000000

 $\overline{9498744} 10000000(1$

9498744

 $\overline{501256} 9498744(18$

9022608

therefore—

 $\sqrt{99} = 9 + \frac{1}{1} + \frac{1}{18} + \frac{1}{1} + \frac{1}{476136} 501256(1$

476136

 $\overline{25120} 476136(18$

452160

 $\overline{23976} 25120(1$

23976

 $\overline{1144} 23976(18$

20592

9, 1, 18, 1, 18,
 $\frac{1}{1}, \frac{1}{18}, \frac{1}{18}, \frac{1}{1}, \frac{1}{99}.$

MISCELLANEOUS EXERCISES.

(Arithmetic, page 254.)

No.

1. $a : b :: c : d$. Ans. $ad=bc$.

2. 5 days : 3 days :: 1 p. work : $\frac{2}{3}$ p. work done by A in 3 days.

From 1 p. work,

take $0\frac{2}{3}$

$0\frac{2}{3}$ p. work done by B in 3 days.

$\frac{2}{3}$ p. work : $\frac{2}{3}$ p. work :: 3 days : $1\frac{1}{2}=7\frac{1}{2}$ days. Ans.

3. $7\frac{1}{2}$ miles : 1 mile :: 60 min. : 8 min. he spent travelling 1 mile going.

$6\frac{1}{2}$ miles : 1 mile :: 60 min. : $9\frac{3}{4}$ min. he spent travelling 1 mile returning.

From $9\frac{3}{4}$ minutes,

take 8

$1\frac{3}{4}$ lost per mile when returning.

$1\frac{3}{4}$ min. : 8 min. :: 1 mile : $6\frac{1}{2}$ miles, the Ans.

4. $1 \times 2 \times 3 \times 4 = 24$. Ans.

6.
$$\begin{array}{r} \text{£}14 \ 10 \ 6 \\ \phantom{\text{£}}15 \\ \hline 315 \overline{)222 \ 17 \ 6} \\ \underline{\phantom{\text{£}}0 \ 13 \ 10} \end{array}$$
 per yard. Ans.

7. 12 m. : 9 m. :: £5 : £3 $\frac{3}{4}$.

£103 $\frac{3}{4}$: £100 :: £600 : £578 6 3 $\frac{1}{8}$. Ans.

8. $13 \times 5 \times 17 = 1105$. Ans.

9. Log. of $20 \times x = \log.$ of $100 = 2$.

Log. of $100 = 2 \div 1.30103$. (Log. of 20) = 1.53724.

[Ans.

No.

5.

$$\text{Log. of } 41 \cdot 2 = 1 \cdot 614897$$

5

$$\begin{array}{r} 2 \overline{) 8 \cdot 074485} \end{array}$$

$$\text{2nd root} = 10895 \cdot 36 = 4 \cdot 0372425 = \text{log. of 2nd root of } 41 \cdot 2^5$$

$$\text{log. of } 3 \cdot 607 = 0 \cdot 557146$$

4

$$\begin{array}{r} 7 \overline{) 2 \cdot 228584} \end{array}$$

$$\text{7th root} = 2 \cdot 08146 = 0 \cdot 318369 = \text{log. of 7th root of } 3 \cdot 607^4$$

$$\text{from } 10895 \cdot 36$$

$$\text{take } \quad 2 \cdot 08146$$

$$\hline 10893 \cdot 27854^{\frac{1}{7}}$$

$$\text{log. of } 10893 \cdot 27854 = 4 \cdot 037158$$

3

$$\begin{array}{r} 18 \overline{) 12 \cdot 111474} \end{array}$$

$$\text{18th root} = 4 \cdot 708101 = 0 \cdot 672859 = \text{log. of 18th root of } 10893 \cdot 27854^3$$

$$1 \cdot 5) 4 \cdot 26$$

$$\begin{array}{r} 2 \cdot 84^{\frac{1}{5}} = 65 \cdot 0537 \overline{) 4 \cdot 708101} \end{array}$$

$$\hline 0 \cdot 0723725 \text{ Ans.}$$

0.

$$\pounds 300$$

$$\pounds 240$$

$$\pounds 240, \text{ in 6 months, gains } 60$$

$$\pounds 300$$

$$60$$

$$\pounds 360 : \pounds 600 :: \pounds 240 : \pounds 400 = \text{B's stock.}$$

$$\pounds 260$$

$$160$$

$$\pounds 60 : \pounds 100 :: 6 \text{ months} : 10 \text{ months.}$$

$$\pounds 160 : \pounds 240 :: 10 \text{ months} : 15 \text{ months} = \text{C's time.}$$

No.

11.

$$\begin{array}{r} 4 \\ 3 \\ 2\frac{1}{2} \end{array}$$

$$\left. \begin{array}{l} 9\frac{1}{2} : 4 :: 300 : 126\frac{1}{2} \\ 9\frac{1}{2} : 3 :: 300 : 24\frac{1}{2} \\ 9\frac{1}{2} : 2\frac{1}{2} :: 300 : 78\frac{1}{2} \end{array} \right\} \text{Answers.}$$

12.

$$\begin{array}{r} 25'73' \\ 25 \\ 4 \overline{)2548} = \frac{637}{2475} \text{ Ans.} \end{array}$$

13. $5^7 = 78125$; then $100\cdot000 - 78125 = 22875$.

$5^8 = 390625$; then $390625 - 100000 = 290625$;
therefore the 7th power is nearer than any other
to 100·000. Ans.

14. $5\cdot5 \div 2\cdot5 = 2\cdot2$ & $2\cdot2 \div (3\cdot5 \div 14) = 2\cdot2 \div \frac{25}{7\frac{1}{2}} = \frac{8\cdot8}{7\frac{1}{2}}$.
 $7\frac{1}{2} \div 7\frac{1}{2} = 1$. $4\cdot743272 \div 3\cdot659308 = 1\cdot296 = 7\frac{1}{2}$. Ans.

15. 11 p. : 1 p. :: 60 m. : 5 m. $27\frac{1}{2}$ sec.—add 1 hour;
then 1 h. 5 m. $27\frac{1}{2}$ sec. Ans.

16. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 3628800$. Ans.

18. 11·685 = present value of a life aged 48 years.

$$\begin{array}{r} 80 \\ \hline 934\cdot8 \end{array}$$

$$\begin{array}{r} \cdot04)1 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 80 \\ \hline \text{from } \pounds 2000 \quad 0 \quad 0 \\ \text{take } 934 \quad 16 \quad 0 \\ \hline \pounds 1065 \quad 4 \quad 0 \text{ Ans.} \end{array}$$

19. $1 \div 1 - \frac{1}{2} = 2$. Ans.

20.

mo.	mo.	£	£						
6	12	::	60	:	120				
15	12	::	100	:	80				

$$\left. \begin{array}{l} 80 \\ 120 \\ 200 \end{array} \right\} \begin{array}{l} \pounds \\ \pounds \\ \pounds \end{array}$$

$$\begin{array}{l} 400 : 800 :: 80 : 160 \text{ C's stock.} \\ 400 : 800 :: 120 : 240 \text{ A's do.} \\ 400 : 800 :: 200 : 400 \text{ B's do.} \end{array}$$

[Answers.]

No.

17. Let $x=2$;
 then $x^2=8$
 $24x=48$
 $\underline{56}$
 ought to be 250
 $\underline{-194}$

$$\begin{array}{r} 2 \\ 90 \\ \hline 180 \end{array}$$

$$194-90=104 \quad) \quad 596 \quad (5 \cdot 73$$

Let $x=5 \cdot 7$;
 then $5 \cdot 7^3=185 \cdot 2$
 $24 \times 5 \cdot 7=136 \cdot 8$
 $\underline{322}$
 ought to be 250
 $\underline{+72}$

$$\begin{array}{r} 5 \cdot 8 \\ 72 \\ \hline 417 \cdot 6 \end{array}$$

$$84-72=12 \quad) \quad 61 \cdot 2$$

Let $x=5 \cdot 1$;
 then $5 \cdot 1^3=132 \cdot 7$
 $24 \times 5 \cdot 1=122 \cdot 4$
 $\underline{255 \cdot 1}$
 ought to be 250
 $\underline{+5 \cdot 1}$

$$\begin{array}{r} 5 \cdot 1 \\ 5 \cdot 2 \\ \hline 26 \cdot 52 \end{array}$$

$$15 \cdot 5-5 \cdot 1=10 \cdot 40 \quad) \quad 52 \cdot 53$$

Again $x=4$;
 then $x^2=64$
 $24x=96$
 $\underline{160}$
 ought to be 250
 $\underline{-90}$

$$\begin{array}{r} 4 \\ 194 \\ \hline 776 \\ 180 \end{array}$$

Let $x=5 \cdot 8$;
 then $5 \cdot 8^3=195 \cdot 2$
 $24 \times 5 \cdot 8=139 \cdot 2$
 $\underline{334 \cdot 4}$
 ought to be 250
 $\underline{+84}$

$$\begin{array}{r} 5 \cdot 7 \\ 84 \\ \hline 478 \cdot 8 \\ 417 \cdot 6 \end{array}$$

Let $x=5 \cdot 2$;
 then $5 \cdot 2^3=140 \cdot 7$
 $24 \times 5 \cdot 2=124 \cdot 8$
 $\underline{265 \cdot 5}$
 ought to be 250
 $\underline{+15 \cdot 5}$

$$\begin{array}{r} 5 \cdot 2 \\ 15 \cdot 5 \\ \hline 79 \cdot 05 \\ 26 \cdot 52 \end{array}$$

$$\underline{5 \cdot 05} \quad \text{Ans.}$$

No.

21.

Let $x=3$;then $x^3=27$

ought to be 100

 -73 $+156 \times 3 = 468$ $-73 \times 4 = 292$ $760 \div 229 = 3.3$ for x .Again, let $x=3.3$;then $x^{3.3} = 51.41573$

ought to be 100

 -48.58427 -35.86595

difference 12.71832

 $-48.58427 \times 3.4 = 165.186518$ $-35.86595 \times 3.3 = 118.357635$ difference $46.828883 \div 12.71832 = 3.6$ for x .Again, let $x=3.3$;then $x^{3.3} = 51.41573$

ought to be 100

 -48.58427 $+ .62040$

sum 49.20467

 $-48.58427 \times 3.6 = 174.903372$ $+ .62040 \times 3.3 = 2.047320$ sum $176.95069 \div 49.20467 = 3.5962$.

Ans.

22. The greatest number denoted by two digits is 99, and the product of 99 by 99 = 9801, just as many digits as are in both factors. Again, the least number expressed by two digits is 10, and the product of 10 by 10 = 100, as many less 1 as are in both factors. Therefore no number expressed by two digits, &c.

Let $x=4$;then $x^4=256$

ought to be 100

 $+156$ -73

sum = 229

Let $x=3.4$;then $x^{3.4} = 64.13405$

ought to be 100

 -35.86595 Let $x=3.6$;then $x^{3.6} = 100.62040$

ought to be 100

 $+ .62040$

No.

$$\begin{array}{r}
 23. \quad 3) 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \\
 \hline
 2) 1 \cdot 2 \cdot 1 \cdot 4 \cdot 5 \cdot 2 \cdot 7 \cdot 8 \cdot 3 \\
 \hline
 2) 1 \cdot 1 \cdot 1 \cdot 2 \cdot 5 \cdot 1 \cdot 7 \cdot 4 \cdot 3 \\
 \hline
 \quad \cdot \quad \cdot \quad \cdot \quad 1 \quad 5 \quad \cdot \quad 7 \cdot 2 \cdot 3
 \end{array}$$

then $5 \times 7 \times 2 \times 3 \times 2 \times 2 \times 3 = 2520$ Ans.

24. $6 \times 3 \times 3 = 54 \div (2 \times 3) + 6 = 4\frac{1}{2}$;
 $6 \times 3 \times 3 = 54 \div (2 \times 6) + 3 = 3\frac{1}{2}$ \therefore 6, $4\frac{1}{2}$, $3\frac{1}{2}$, 3—the series. Ans.

25. The circle of circulates should contain but 6 places.
 See Arithmetic page 167.

27. Inter the indices fractionally, and their continual product will give the required index; thus—
 $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$, therefore $10^{\frac{1}{16}}$ is the Answer.

28. £200)645

£3.2251, this found in the table under 5 per cent., and opposite to it in the time column, will be found 24 years, the time required.

29. $(1 \times 4 \times 6 \times 9) + 3 = 219 \div 15 = 14$ and 9 over.
 should be 12

$$\begin{array}{r} -3 \end{array}$$

$(2 \times 4 \times 6 \times 9) + 3 = 435 \div 15 = 29$ and 0 over.
 should be 12

$$\begin{array}{r} -12 \end{array}$$

$$219 \times 12 = 2628$$

$$435 \times 3 = 1305$$

difference $1323 \div (12 - 3) = 9 = 147$ Ans.

30. $30 \times 29 \times 28 \times 27 \times 26 = 17100720$ Ans.

31. $\overline{3+5} \times \overline{3+4} = 3^2 + (\overline{5+4}) \times 3 + \overline{5 \times 4}$

$(3-5) \times (3-4) = 3^2 - (\overline{5+4}) \times 3 + \overline{5 \times 4}$

difference $2(\overline{5+4}) \times 3$, and $(3-4) \times (3-4) = 3^2 - 4^2$.

[Ans.]

No.

26. Let x =correct time; then $x-3$ =time £600 bears interest at, suppose, 4 per cent.

$100 \times 12 : 600 \times (x-3) :: £4 : 2x-6$ =interest on £600; and $8-x$ =time £400 bears discount;

then 12 mo. : $8-x$ mo. :: $£4 : \frac{8-x}{3}$; also

$100 + \frac{8-x}{3} : 400 :: \frac{8-x}{3} : \frac{3200-400x}{308-x}$ = discount on £400.

But by the question, the interest on £600 should =the discount on £400, therefore—

$$2x-6 = \frac{3200-400x}{308-x}, \text{ or } x-3 = \frac{1600-200x}{308-x}.$$

Multiply both sides of the equation by $308-x$, and then by transposition we have $x^2-511x=-2524$, or $x^2-511x+2524=0$. The root by Horner's method, as follows, gives the required time :—

511	2524	[4.892=4 mo. 26 $\frac{2}{3}$ days, nearly.
4	2060	Ans.
515	46400	
4	41584	
5190	481600	
8	468621	
5198	1297900	
8	1041364	
52060	256536	
9		
52069		
9		
520680		
2		
520682		

No.

32. $(3+4)^{\frac{1}{2}}=7^{\frac{1}{2}}$, \log of $7=0.8450980$

$$\begin{array}{r} 5 \\ 9 \overline{)4.2254900} \\ \log. \text{ of } 7^{\frac{1}{2}} = 0.4694988 \end{array}$$

$$\begin{array}{r} \log. 7 = 0.8450980 \\ 4 \\ 9 \overline{)3.3803920} \\ \log. \text{ of } 7^{\frac{1}{2}} = 0.3755991 \\ 0.4694988 \\ \log. \text{ of } 7 = 0.845098 \end{array}$$

therefore $5^x=7$, and $\log. \text{ of } 5^x = \log. \text{ of } 7 = 0.845098$;

$$\begin{array}{l} \text{then } \log. \text{ of } 7 \\ \log. \text{ of } 5 = x; \end{array}$$

$$\begin{array}{l} \text{then } \log. \text{ of } 7 = 0.845098 \\ \log. \text{ of } 5 = 0.698970 = 1.2090619 \text{ Ans.} \end{array}$$

34. $3\frac{1}{2}$ acres : 10 acres \therefore 12 oxen : 36 oxen, which 10 acres will keep in 4 weeks.

9 weeks : 4 weeks \therefore 36 oxen : 16 oxen, to be kept in 9 weeks.

The growth of grass on 10 acres in 5 weeks will be sufficient to feed 5 oxen for 9 weeks; that is, $21-16=5$ oxen.

18 weeks : 9 weeks \therefore 5 oxen : $2\frac{1}{2}$ oxen, in 18 weeks. 18 weeks $-$ 4 weeks $=$ 14 weeks, and 9 weeks $-$ 4 weeks $=$ 5 weeks;

then 5 weeks : 14 weeks \therefore $2\frac{1}{2}$ oxen : 7 oxen.

7 oxen $+$ 8 oxen $=$ 15 oxen, which 10 acres will keep in 18 weeks.

Lastly, 10 acres : 24 acres \therefore 15 oxen : 36 oxen.

[Ans.

No.

33. $\frac{\pounds 1.00}{.06} = 16.6' = \text{the perpetuity of } \pounds 1,$
 $\frac{9.417}{7.249} = \text{value of an. of } \pounds 1 \text{ on a single life of 50 yrs.}$
 $\frac{7.249}{7.249} = \text{their difference.}$

then $\frac{7.249}{1 \times 16.6} = \left\{ \begin{array}{l} .410336, \text{ and this by } 1000 = \text{£}410.336 = \\ \text{the value at the expiration of the life.} \end{array} \right.$

amt. of £1 in 10 years, $= \frac{\text{then } \text{£}410.336}{1.7908} = \left\{ \begin{array}{l} \text{£}229 \ 3 \ 8 \\ \text{Ans.} \end{array} \right.$

35.
$$\begin{array}{r} 12 \overline{)1006} \\ 12 \overline{)83} - 10, \text{ therefore } 6 \text{ ba, Ans.} \\ \underline{6-11} \end{array}$$

36. $30 \div 2 = 15 = \frac{1}{2}$ sum;
 $15 = 225$ from
 216 take

 $9 = 2$ nd power of $\frac{1}{2}$ difference.
 $\sqrt{9} = 3 = \frac{1}{2}$ difference;
then $15 + 3 = 18$ }
and $15 - 3 = 12$ } Ans.

37. By a process similar to that for finding the greatest common measure (paragraph 205), the several quotients will be 2, 3, 3, 3, 1, 1, 3, 6, 3, 1, 1, 1, 3, 1, 11, 5, &c., and these give the convergents, $\frac{1}{2}, \frac{3}{2}, \frac{10}{7}, \frac{18}{11}, \frac{29}{18}, \frac{47}{29}, \frac{114}{77}, \frac{191}{118},$ &c. Ans.

38. Suppose £100; Suppose £250;

25 in. for 5 years.	62½ in. for 5 years.
20 discount.	50 discount.
<u>5</u> =difference.	<u>12½</u> =difference.
should be $\frac{12}{-7}$	should be $\frac{12}{+\frac{1}{2}}$

then $7 \times 250 = 1750$
 $\frac{1}{2} \times 100 = 50$
 $7 + \frac{1}{2} = 7.5) \overline{1800.0}$
 $\underline{\hspace{1.5cm}} \text{£240 Ans.}$

No.

39. $\sqrt{17}=4.123105$, this first divided by 1000000, the denominator of the decimal part, and proceeding by the rule for finding the greatest common measure, we find the following quotients, viz.—4, 8, 8, 8, 3, 2, 1, 1, 1, 2, 1, 3, which, converted into convergents, give $\frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}, \frac{1}{1}$ for the Ans.

40. The 2nd power of a number=2nd powers of any 2 parts thereof+twice their product.

The side of 1st square+1=side of 2nd square.

The 1st square left 50. The 2nd square wanted 11.

Then $50+11=61$ =difference of 2 squares, but this=the 2nd power of 1, + twice the product of 1 by the side of 1st square.

$61-1=60$, the $\frac{1}{2}$ of which=30=product of the parts. As 1 is one of these parts, $30=30$ =the other part, viz.—the side of first square.

$30^2=900$, and $900+50=950$ =number of men.

Otherwise—

[Ans.

Let x =side of 1st square, and $x+1$ =side of 2nd square; then $x^2+50=(x+1)^2-11$, or $x^2+50=x^2+2x+1-11$; $\therefore 2x=60$, and $x=30$, and $x^2=900$; x^2+50 950 the number of men. Ans.

41. $\frac{2}{3} \times \frac{4}{5} \times \frac{3}{4} = \frac{2}{5}$, $1 \div \frac{2}{5} = \frac{5}{2} = 1\frac{1}{2}$. Ans.

42.
$$57 \left\{ \begin{array}{l} 60 \overline{) 3+9=12} \\ 54 \overline{) 3} = 3 \\ 48 \overline{) 3} = 3 \end{array} \right. \quad 18 : 24 :: \left\{ \begin{array}{l} 12 : 16 \\ 3 : 4 \\ 3 : 4 \end{array} \right\} \text{Ans.}$$

18

43. $.007625 : 3.05 :: £1 : £400$. Ans.

44. $10=a$, $11=b$, $12=c$, $13=d$, $14=e$, $15=f$, $16=g$, $17=h$, $18=i$, $19=j$. $h \ 3 \ a \ h \ g$. Proof.

$$\begin{array}{r} 20)2748356 \\ 20)137417-16 \ g \\ 20)6870-17=h \\ 20)343-10=a \\ \hline h=17-3 \end{array}$$

$$\begin{array}{r} 20 \\ 343 \\ 20 \\ 6870 \\ 20 \\ \hline 137417 \\ 20 \\ \hline 2748356 \end{array}$$

$3ahg$ Ans.

No.

45. This question is unlimited, and therefore will admit of many answers besides those given in the work ; as example—

Suppose horses, 20 at 31 = 620

„ oxen, 50 at 21 = 1050

1670

should be 1770

error -100

Again, suppose horses, 20 at 31 = 620

„ oxen, 60 at 21 = 1260

1880

should be 1770

error +110

	20	50	20	60
	110	110	100	100
	<u>2200</u>	<u>5500</u>	<u>2000</u>	<u>6000</u>
+110	2000			5500
-100	210) <u>4200</u>		210) <u>11500</u>	
<u>210</u>	20 horses.		54 $\frac{1}{11}$ oxen.	

Proof—20 horses $\times 31 = 620$

54 $\frac{1}{11}$ oxen $\times 21 = 1150$

crowns, 1770

Thus showing that many numbers besides those given in the work will answer the conditions of the question.

46. $3 \cdot 386355 \times 1000 = 3386 \cdot 355 =$ amount of £1000 in 25 years at 5 per cent. per annum.

$47 \cdot 727098 + 100 = 4772 \cdot 7098 =$ amount of £100 annuity in 25 years, at 5 per cent. ;

then $4772 \cdot 7098 + 3386 \cdot 355 = £8159 \cdot 0648$. Ans.

48. $1 \cdot 30103 (\log. \text{ of } 20) \div 1 (\log. \text{ of } 10) = 1 \cdot 30103$.

49. $\frac{5+9}{2} = 7 =$ middle term.

$5 + 7 + 9 = 21$; then $21 : 63 :: \left\{ \begin{array}{l} 5 : 15 \\ 7 : 21 \\ 9 : 27 \end{array} \right\}$ Ansrs.

No.

47.	$ \begin{array}{r} 21ab7. \\ \underline{12} \\ 25 \\ \underline{12} \\ 310 \\ \underline{12} \\ 3731 \\ \underline{12} \\ 44779 \text{ numer.} \end{array} $	$ \begin{array}{r} ba956. \\ \underline{12} \\ 142 \\ \underline{12} \\ 1713 \\ \underline{12} \\ 20561 \\ \underline{12} \\ 246738 \text{ denom.} \end{array} $	$\left\{ \begin{array}{l} \text{then} \\ \frac{44779}{246738} \text{ Ans.} \end{array} \right.$
-----	--	--	---

50.	$ \begin{array}{r} x^2 - 6x = 40 \\ x^2 - 6x + 9 = 49 \\ x - 3 = 7 \\ x = 10. \text{ Ans.} \end{array} $	$ \begin{array}{r} \text{By Arithmetic—} \\ -6 \quad 4)40 \\ +10 \quad 40 \\ \hline 4 \end{array} $	$[10 \text{ Ans.}]$
-----	---	---	---------------------

51. $\frac{461.475}{264.583} = \text{amount,}$ } $= 1.744157480;$
 $\frac{\quad}{\quad} = \text{principal,}$ }

then $\sqrt[3]{1.744157480} = 1.32066554,$

and $\sqrt[3]{1.32066554} = 1.149202136;$

then $\sqrt[3]{1.149202136} = 1.047447;$

from $1.047447 = \text{amount of } £1 \text{ in 1 year.}$

take 1.

$.047447 = \text{interest of } £1 \text{ in 1 year.}$

$.047447 \times 100 = 4.7447, \text{ or } 4\frac{3}{4} \text{ nearly} = \text{rate per cent. Ans.}$

52. 1000, &c., *ad infinitum*. Ans.

53. $\frac{1}{2} \div \frac{1}{4} = \frac{4}{2} = 2, \therefore 1 - \frac{1}{2} = \frac{1}{2} = \text{ratio. Ans.}$

54. From log. of $10 = 1.$

take log. of $5 = .69897$

$\frac{.30103}{.69897} \div .021189 (= \log. 1.05)$
 $= 14.206896 \text{ years. Ans.}$

56. $\sqrt[3]{\frac{9}{10}} = \frac{9^{\frac{1}{3}}}{10^{\frac{1}{3}}}$; and if we multiply both terms of this fraction by $9^{\frac{1}{3}}$, we have $9^{\frac{1}{3}} \times 9^{\frac{1}{3}} = 9$, and $10^{\frac{1}{3}} \times 9^{\frac{1}{3}} = 10^{\frac{1}{3}} \times 81^{\frac{1}{3}} = 810^{\frac{1}{3}}$; therefore $\sqrt[3]{\frac{9}{10}} = \frac{9}{810^{\frac{1}{3}}} \text{ Ans.}$

No.

55. From $3a'5c'$ take $3a$

$$\begin{array}{r} 3a22 \\ 3a22 \\ \hline eeo0 \\ 15 \\ \hline 224 \\ 15 \\ \hline 3360 \\ 15 \end{array}$$

$$\begin{array}{r} 3a22 \\ 15 \\ \hline 55 \\ 15 \\ \hline 827 \\ 15 \\ \hline 12407 \end{array}$$

$12407 = \text{numer. } \therefore 12407 \text{ Ans.}$

$50400 = \text{denominator.}$

58. $10 - 6 = 4$. $4 : 73 :: 1 : \frac{73}{4} = 18\frac{1}{4}$ days B overtakes A.
 $16 - 10 = 6$. $6 : 73 :: 1 : \frac{73}{6} = 12\frac{1}{6}$ days C overtakes B.
 $18\frac{1}{4} - 12\frac{1}{6} = 6\frac{1}{12}$, and $18\frac{1}{4} \div 6\frac{1}{12} = 3$; $12\frac{1}{6} \div 6\frac{1}{12} = 2$.
 Therefore $6\frac{1}{12} \times 3 \times 2 = 36\frac{1}{2}$ days all meet. Ans.

59. Let $x = \text{the number}$, then $x^2 = 7x + 9$, or $x^3 - 7x - 9 = 0$, or $x^3 - 0x^2 - 7x - 9 = 0$; then

0	-7	-9 (31409238. Ans.
3	9	6
3	2	-3000
3	18	2091
6	2000	-909000
3	91	888144
90	2091	-2086680000
1	92	20328550929
91	218300	-527449071000
1	3736	451918937688
92	222036	-75530133312000
1	3752	&c.
930	2257880000	
4	847881	
934	2268727881	
4	847962	
938	225957584300	
4	1884544	
94200	225959468844	
9	1884548	
94209	22596135339200	
9	&c.	
94218		
9		
942279		
&c.		

By contracting this process the answer will be found as above.

No.

57. $\sqrt[3]{63} = 7.9372539$; then $10000000 \div 9372539 = 1 + 627461 \text{ rem.}$ $9372539 \div 627461 = 14 + 588085 \text{ rem.}$ $627461 \div 588085 = 1 + 39376 \text{ rem.}$ $588085 \div 39376 = 14 + 34821 \text{ rem.}$ $39376 \div 34821 = 1 + 4555 \text{ rem., \&c. \&c.}$; so that if the decimal of the root were carried far enough, and divided as above, the quotients would be 1 and 14 alternately; therefore the root of 63 = $7 + \frac{1}{1} + \frac{1}{14} + \frac{1}{14} + \frac{1}{1} + \frac{1}{14}, \&c.$

60. Log. of 97 = 1.986772
216

$\overline{429.142752} \div 25 = 17.165710 = \log.$ of $97^{\frac{1}{25}}$, but as the number of integral figures in the number itself exceeds the number of units in the log. by 1, therefore $\sqrt[25]{97^{216}}$ contains 18 integral places. Ans.

61. $.69897 (= \log. \text{ of } 5) - .60206 (= \log. \text{ of } 4) = .09691 (= \log. \text{ of } \frac{5}{4})$; then $1.7363965 (= \log. \text{ of } 54\frac{1}{2}) \div .09691 = 17.9176 = \text{Ans.} = \text{value of } x$.
62. Let $x = \text{capital}$; then $2x - 240 = \text{1st year's net worth.}$
 $(2x - 240) \times 2 = 4x - 480 = \text{2nd year realized.}$ $(4x - 720) \times 2 = 8x - 1440 = \text{2nd year's worth; and } 8x - (1440 + 240) = 8x - 1680 = \text{3rd years' worth.}$
 $(8x - 1680) \times 2 = 16x - 3360$; and $16x - (3360 + 240) = 16x - 3600 = \text{4th years' worth.}$

$$16x - 3600 = \frac{3x}{4}$$

$$64x - 14400 = 3x$$

$$(64x - 3x) = 61x = 14400$$

$$\text{and } x = \frac{14400}{61} = \text{£}236 \frac{1}{61} \text{ } 3\frac{4}{61} \text{ Ans.}$$

63. $256 = 2^8$, therefore $256 \times \frac{2-1}{2} = 128$ numbers less than 256, and prime to it. Ans.

64. $7a \cdot a3b$ 2103. a63113

$$31 \cdot ba9$$

$$\frac{12}{12}$$

$$\frac{5b18b\bar{3}}{25}$$

$$670732$$

$$\frac{12}{12}$$

$$72b571$$

$$\frac{300}{300}$$

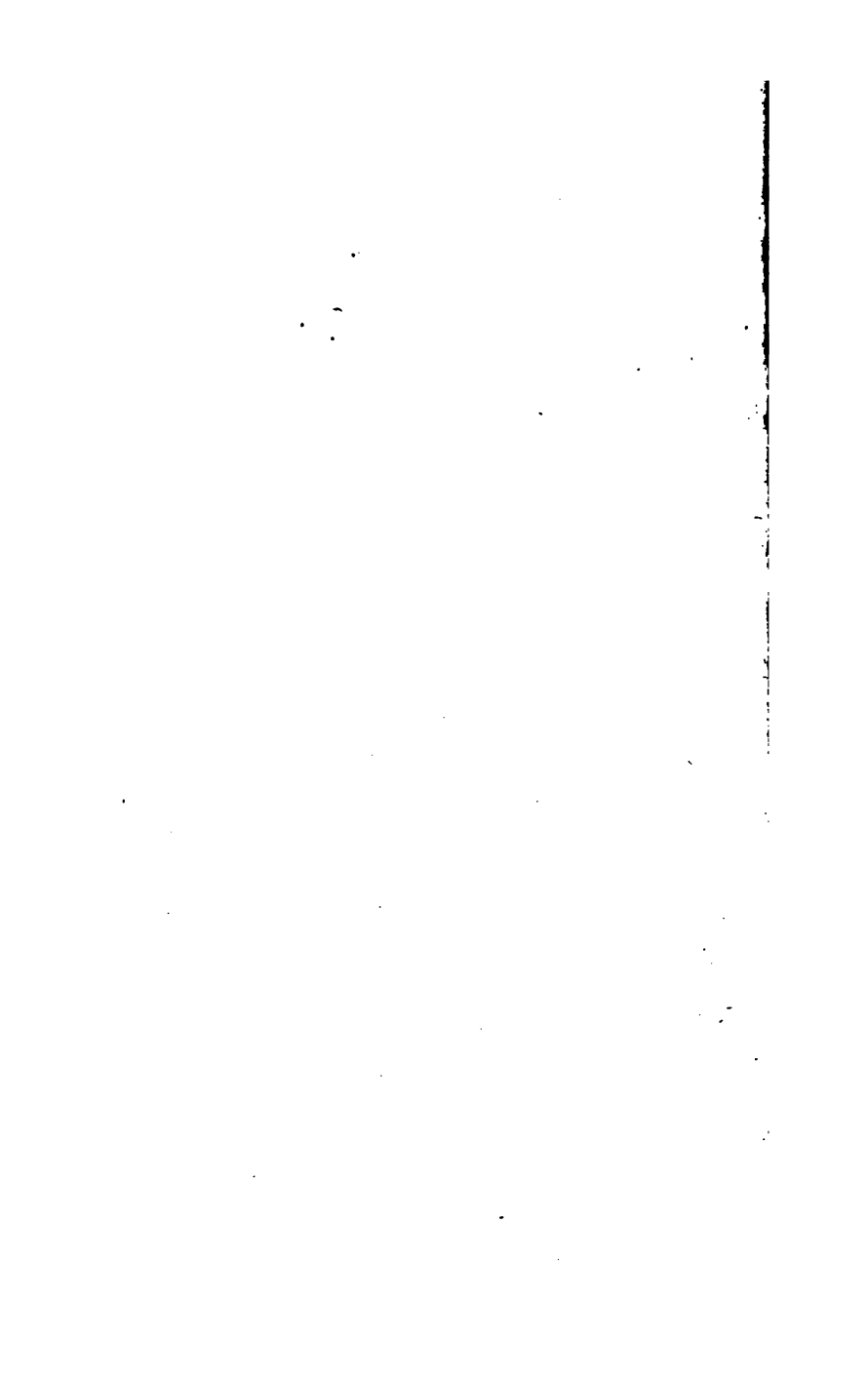
$$7aa3b$$

$$\frac{12}{12}$$

$$1b86b9$$

$$\frac{3603, 10 \text{ in., } 6 \text{ part, } 3 \text{ sec. } 1''' 1''' 3''' \text{ Ans.}}{3603, 10 \text{ in., } 6 \text{ part, } 3 \text{ sec. } 1''' 1''' 3''' \text{ Ans.}}$$

$$\frac{2103 \cdot a63113}{2103 \cdot a63113}$$



1

2





